Laboratory 5

Numerical simulations of the control system
Basic structures of PID controllers

1. Purpose of the exercise:

- numerical simulations of the control system using MATLAB/Simulink environment
- learning the PID controller family structures
- analyses of the control system operating characteristics

2. Basic structures of the linear controllers

Block diagram of the **PID** controller structure is presented in Fig. 1. Transfer functions of the PID controller family:

- Proportional controller (**P**): $G_R(s) = K_r$
- Integral controller (**I**): $G_R(s) = \frac{1}{T_i s}$
- Proportional-Integral controller (**PI**): $G_R(s) = K_r\left(1 + \frac{1}{T_i s}\right)$
- Proportional-Derivative controller (**PD**): $G_R(s) = K_r(1 + T_d s)$
- Proportional-Integral-Derivative controller (**PID**): $G_R(s) = K_r\left(1 + \frac{1}{T_i s} + T_d s\right)$

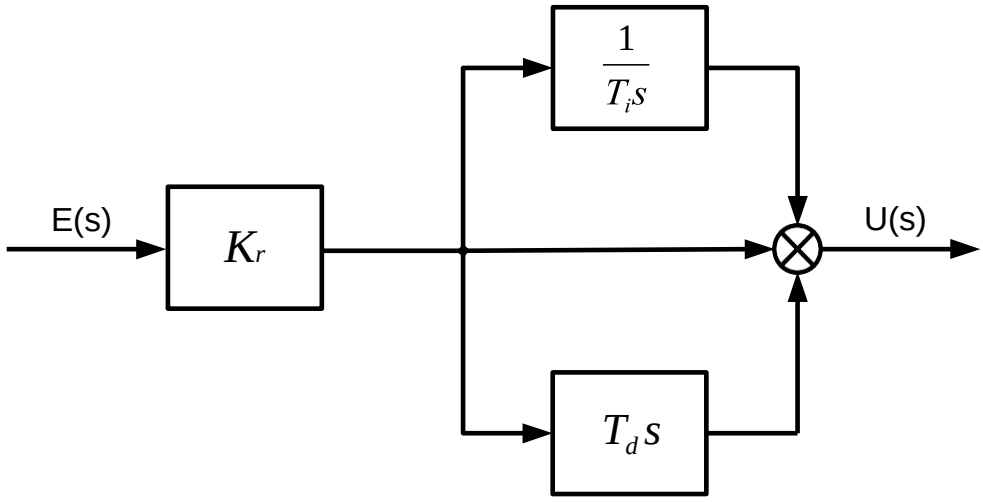


Fig. 1. General structure of the PID controller

Using MATLAB/Simulink, build the control system according to Fig 2:

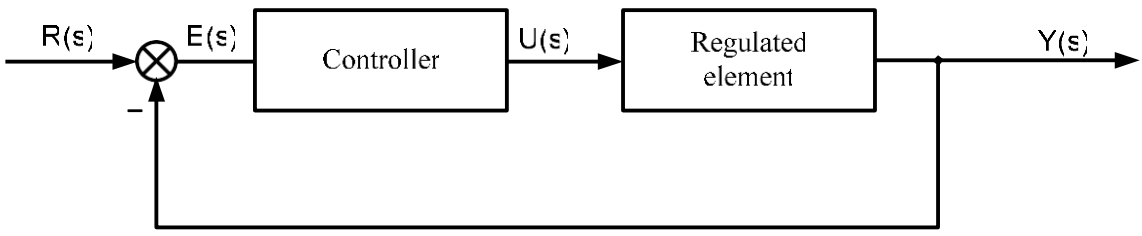


Fig. 2. Block diagram of the feedback control system

While constructing the above-depicted system model consider:

- input signal: ***r*(t)** (step function)
- output signal: ***y*(t)**
- regulated element:

		PC no. 1/6/11	PC no. 2/7/12	PC no. 3/8/13	PC no. 4/9/14	PC no. 5/10/15
1st order inertial element	**K**	**1**	**1**	**1**	**1**	**1**
	T	**1**	**2**	**3**	**4**	**5**
2nd order oscillatory element	**K**	**1**	**2**	**3**	**4**	**5**
	ω	**1**	**1**	**1**	**1**	**1**
	ς	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**

Build the following feedback control systems:

1. control system with P controller,
2. control system with I controller,
3. control system with PI controller,
4. control system with PD controller,
5. control system with PID controller.

Assume simulation time range of 100 s.

Find an influence of controllers parameters (K_r, T_i, T_d) on operating characteristics of the analysed control system using following indexes: steady-state error $\boldsymbol{e_{SS}}$, overshoot $\boldsymbol{\kappa}$, settling time $\boldsymbol{t_S}$ (see *Appendix*).

3. Design of the control system

Build the control system for the elements given below and select a controller so that the settling time t_S is shortest (for steady-state error $e_{SS} = 0$):

a) $G_1(s) = \frac{1}{s^2+1}$, b) $G_2(s) = \frac{10}{s^2+s+1}$, c) $G_2(s) = \frac{0.228}{0.18s^2+1.18s+1}$

	PC no. 1 / 6 / 11	PC no. 2 / 7 / 12	PC no. 3 / 8 / 13	PC no. 4 / 9 / 14	PC no.5 / 10 / 15
Transfer function	**c**	**b**	**a**	**b**	**c**

Build the control system for the elements given below and select a controller so that the overshoot κ is smallest (for steady-state error $e_{SS} = 0$):

a) $G_1(s) = \frac{2.5}{2s^2+2s+1}$, b) $G_2(s) = \frac{1}{s^3+2s^2+2s+1}$

	PC no. 1 / 6 / 11	PC no. 2 / 7 / 12	PC no. 3 / 8 / 13	PC no. 4 / 9 / 14	PC no.5 / 10 / 15
Transfer function	**a**	**b**	**a**	**b**	**a**

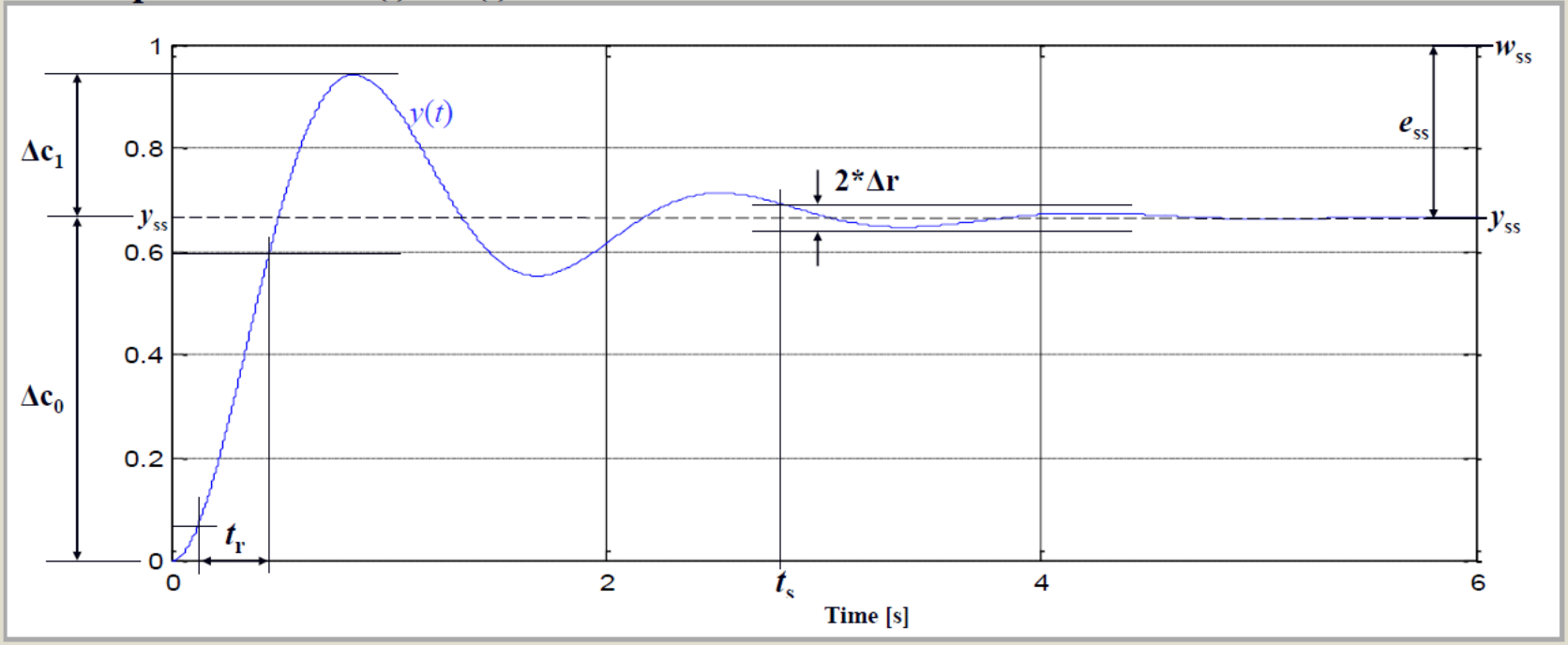
On the basis of command input *r*(t) and output *y*(t) signals time runs, determine for the analysed control system:

- steady-state error $\boldsymbol{e_{SS}}$
- overshoot $\boldsymbol{\kappa}$
- settling time $\boldsymbol{t_S}$ for 5% tolerance of steady-state output $\boldsymbol{y_{SS}}$

References:

[1] G.F. Franklin, J.D. Powell, E. Emami-Naeini “Feedback control of dynamic systems”, Prentice Hall, New York, 2006.
[2] K. Ogata “Modern control engineering”, Prentice Hall, New York, 1997.
[3] R.H. Cannon “Dynamics of physical systems”, Mc-Graw Hill, 1967 (available in Polish as: R.H. Cannon “Dynamika układów fizycznych”, WNT, Warszawa, 1973).
[4] J. Kowal “Podstawy automatyki”, v.1 and 2, UWND, Kraków, 2006, 2007 (in Polish).
[5] W. Pełczewski “Teoria sterowania”, WNT, Warszawa, 1980 (in Polish).
[6] Brzózka J., Ćwiczenia z Automatyki w MATLABIE i Simulinku, Wydawnictwo Mikon, Warszawa 1997 (in Polish).
[7] Zalewski A., Cegieła R., MATLAB: obliczenia numeryczne i ich zastosowania, Wydawnictwo Nakom, Poznań 1996 (in Polish).
[8] MATLAB/Simulink documentation: http://www.mathworks.com/help/

Setpoint value: $w(t) = 1(t)$



Steady-state error: $e_{ss} = w_{ss} - y_{ss}$ where: $y_{ss} = \lim_{t\to\infty} y(t)$ $w_{ss} = \lim_{t\to\infty} w(t)$

Rise time: t_r – time needed for response $y(t)$ change from 10% to 90% (or from 5% to 95%) of the steady-state value y_{ss}

Settling time: t_s – time needed for response $y(t)$ to reach y_{ss} neighbourhood of radius Δr
$\Delta r = 3 \div 5\% \; y_{ss}$

Overshoot: $\kappa = \Delta c_1 / \Delta c_0 \cdot 100\%$