Laboratory 4

Parametric synthesis of control systems

Purpose of the exercise:

Parametric synthesis of the control system, use of the Ziegler-Nichols method.
MATLAB/Simulink environment is used.

1. Operating characteristics of the selected control system

1.1. Simulate the feedback system presented in Fig. 1 with MATLAB/Simulink software:

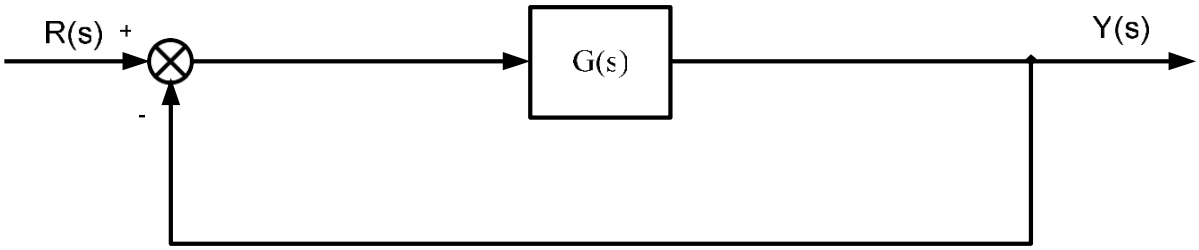


Fig. 1. Block diagram of the feedback system (P control system with controller gain $K_r = 1$)

Constructing the above-depicted system consider:

- input signal: ***r*(t)** (step function)
- transfer function:

$$G(s) = \frac{1.5}{(2s + 1)(3s + 1)(4s + 1)(5s + 1)}$$

- simulation time: 100 s

1.2. On the basis of command input *r*(t) and output *y*(t) signals time runs, determine for the analyzed control system:

- steady-state error $\boldsymbol{e_{ss}}$,
- overshoot $\boldsymbol{\kappa}$,
- settling time $\boldsymbol{t_s}$ assuming Δ**r**=5% of steady-state output tolerance.

2. Controller selection

2.1. Definition of the target of the control system parametric synthesis based on the following demands:

- steady-state error elimination (obligatory),
- reduction of overshoot to 0% / 10% / 20%,
- reduction of settling time.

2.2. Selection of the controller type should be realized based on the data from Table 1:

Term	**Term implementation effect**	**Controller type**
P	modification of steady-state error, modification of overshoot, a small modification of settling time	P / PI / PID (PD)
I	steady-state error elimination, modification of overshoot, an increase in settling time	PI / PID (I)
D	reduction of both: settling time and overshoot, noise amplification	PID (PD)

3. Ziegler-Nichols method

3.1. Simulate the system presented in Fig. 2 with MATLAB/Simulink software:

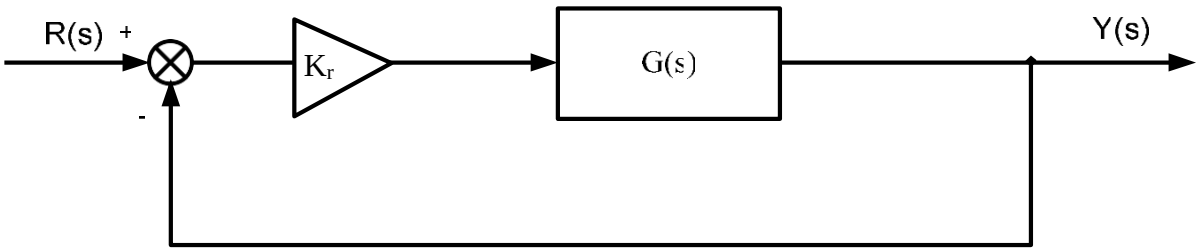


Fig. 2. Block diagram of the control system for the Ziegler-Nichols method

Constructing the above-depicted control system model consider:

- input signal: ***r*(t)** (step function)
- transfer function:

$$G(s) = \frac{1.5}{(2s + 1)(3s + 1)(4s + 1)(5s + 1)}$$

- simulation time: 100 s

The control system should be led to the boundary of its stability by increasing gain K_r (see Fig. 2). A value of K_r, at which the control system reaches the stability boundary is called critical gain (K_{cr}). The controller settings are calculated based on the critical gain (K_{cr}) and oscillation period (T_{osc}) using the formulas from Table 2.

Table 2:

Controller	**K_r**	**T_i**	**T_d**
P	0,5 K_{cr}	-	-
PI	0,45 K_{cr}	0,83 T_{osc}	-
PID	0,6 K_{cr}	0,5 T_{osc}	0,125 T_{osc}

Calculated settings should be written into the Table 3:

Controller type	**Controller settings**			**Operating characteristics**			
	K_r	**T_i**	**T_d**	**e_{ss}**	**κ**	**Δr**	**t_s**
P controller		----	----			5%	
PI controller			----			5%	
PID controller						5%	

4. Tasks

4.1. Find the controller settings for the transfer function G(s) (see Fig. 2) using Ziegler-Nichols method:

$$G(s) = \frac{a}{(s + 1)(3s + 1)(2s + 1)^3}$$

Constructing the controller consider:

- input signal: ***r*(t)** (step function)
- simulation time: 100 s
- Table 4. Value of the parameter *a*:

	PC 1 / 6 / 11	PC 2 / 7 / 12	PC 3 / 8 / 13	PC 4 / 9 / 14	PC 5 / 10 / 15
a	**1**	**1.2**	**1.4**	**1.6**	**1.8**

Calculated settings and operating characteristics indexes should be written into Table 3.

4.2. Find the controller settings for the mechanical system below. Determine its transfer function assuming zero initial conditions. Assume the target of the control system parametric synthesis and controller selection criteria according to sections *2.1* and *2.2*, respectively (assume 10% overshoot target). Use general PID controller family expert tuning rules, as given below:

– *while increasing proportional (P) term, e_{ss} is decreasing, and κ is increasing,*

– *while increasing integral (I) term, κ and t_s are both increasing (e_{ss} is cleared for nonzero I-term),*

– *while increasing derivative (D) term, κ and t_s are both decreasing.*

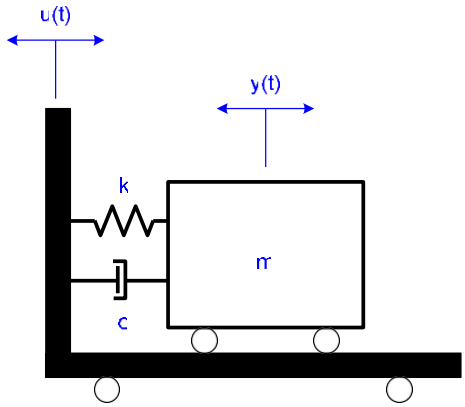


Fig. 3. Mechanical system

While constructing the above-depicted system model consider:

- input signal: displacement **u(t)** (step function)
- output signal: displacement **y(t)**
- Table 5. Value of the parameters:

Paramete	Unit	PC 1 / 6 / 11	PC 2 / 7 / 12	PC 3 / 8 / 13	PC 4 / 9 / 14	PC 5 / 10 / 15
c	Ns/m	**1**	**1,5**	**2**	**2,5**	**3**
k	N/m	**3**	**2,5**	**2**	**1,5**	**1**
m	kg	**1**	**1,5**	**2**	**2,5**	**3**

Calculated settings and operating characteristics indexes should be written into Table 3.

References:

[1] G.F. Franklin, J.D. Powell, E. Emami-Naeini "Feedback control of dynamic systems", Prentice Hall, New York, 2006.
[2] K. Ogata "Modern control engineering", Prentice Hall, New York, 1997.
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[4] J. Kowal "Podstawy automatyki", v.1 and 2, UWND, Kraków, 2006, 2007 (in Polish).
[5] W. Pełczewski "Teoria sterowania", WNT, Warszawa, 1980 (in Polish).
[6] Brzózka J., Ćwiczenia z Automatyki w MATLABIE i Simulinku, Wydawnictwo Mikon, Warszawa 1997 (in Polish).
[7] Zalewski A., Cegieła R., MATLAB: obliczenia numeryczne i ich zastosowania, Wydawnictwo Nakom, Poznań 1996 (in Polish).
[8] MATLAB/Simulink documentation: http://www.mathworks.com/help/