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Thermodynamics

Laboratory

Laboratory Exercise Manual:

Analysis of basic thermodynamic processes

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THERMODYNAMIC PROCESSES (TRANSFORMATIONS)

As it has been shown in subsection 1.2.5, a process is a change of parameters of state of a thermodynamic system. The process is defined by the succession of states, through which the system passed. A reason for coming into being of any thermodynamic process are the influences between some thermodynamic system and its surroundings which cause changes of state of the thermodynamic agent in this system. However until now we did not consider the path of passage between two states of the thermodynamic agent (history of the system), which e.g. absorbs a heat and performs a work. We only ascertained that the thermodynamic parameters in one state were other than in the second one. We also ascertained that a changes of values of functions of state (e.g. an increasement of internal energy) depend only on the initial and final equilibrium states, instead such quantities as the heat and work depend also on the path of the change of parameters of state of the thermodynamic agent. In thermodynamics the **path** is understood as the specification of a series of states through which the system passes.

The process of changes of value of thermic parameters of state of a thermodynamic agent (*p, V, T*), which takes place between two of its distinguished states (the initial and final state of equilibrium) we call a thermodynamic transformation (or thermodynamic process).

In thermodynamics a special significance have the quasi–static processes. When the values of parameters of state of system change infinitely slowly, coming through the consecutive states of the permanent equilibrium, we call this transformation the quasi–static transformation or the equilibrium transformation. During this transformation the system internally must be all the time infinitesimally close to the state of equilibrium, that is to say, the path of the equilibrium transformation is a series of consecutive equilibrium steps.

The further parts of these lectures refer exclusively to the equilibrium processes.

An agent in a thermodynamic system can be (theoretically) brought over from one state to the second one by way of infinite number of transformations. Of course in practice only a limited number of the thermodynamic transformations is applied, the so called "characteristic" transformations. These are the transformations for which exists some characteristic condition (this can be e.g. a fixed value of some of the three parameters of state; p, v or T, the definite dependency between two parameters of state or the limitation enforced on the influences between the system and its surroundings). Of course, these transformations represent real processes only approximately, nevertheless they allow to foresee various overall effects even though we do not know the detailed description of the process.

The function binding together the parameters of state and describing the path of its changes in a given process we term a **transformation equation.**

Generally, thermodynamic transformations can be divided into:

- **Transformations without a dissipation of energy** (commonly qualified as frictionless). In these transformations the entire heat input to (or led out from) the system is equal to the heat absorbed (or given back) by the thermodynamic agent inside the system.
- **Transformations with the dissipation of energy** (commonly qualified as "with friction"). In these transformations the entire heat absorbed by the thermodynamic agent in the system is equal to the algebraic sum of the heat supplied into the system and the heat of friction.

If the transformation is the equilibrium transformation and takes place without the dissipation of energy, then it is a reversible transformation, instead if in the transformation it is necessary to take into consideration the friction, then it becomes an irreversible transformation.

CHARACTERISTIC PROCESSES (TRANSFORMATIONS) OF IDEAL AND SEMIIDEAL GASES

Isothermal Process

The isotherm (the isothermal process) we call the transformation occuring at the constant temperature of a gas. From the equation of state of an ideal gas (the Clapeyron equation) results that for $T = const$:

$$pv = RT = const \tag{6.1}$$

that is to say, for two different states of the gas:

$$p_1 v_1 = p_2 v_2 \tag{6.1a}$$

or:

$$\frac{p_1}{p_2} = \frac{v_2}{v_1} \tag{6.1b}$$

In case when the transformation taking place in the closed system is considered (the constant mass of the agent), the specific volumes occurrent in the above formulas can be replaced with total volumes, because:

$$\frac{V_2}{V_1} = \frac{M.\cdot v_2}{M \cdot v_1} = \frac{v_2}{v_1}$$

The formulas (6.1a) and (6.1b) are the basic equations of the isothermal transformation, expressing also the so called Boyle-Mariott's law – see (2.3). The isotherms for different values of temperatures in the p–v coordinate system are shown in figure 6.1.

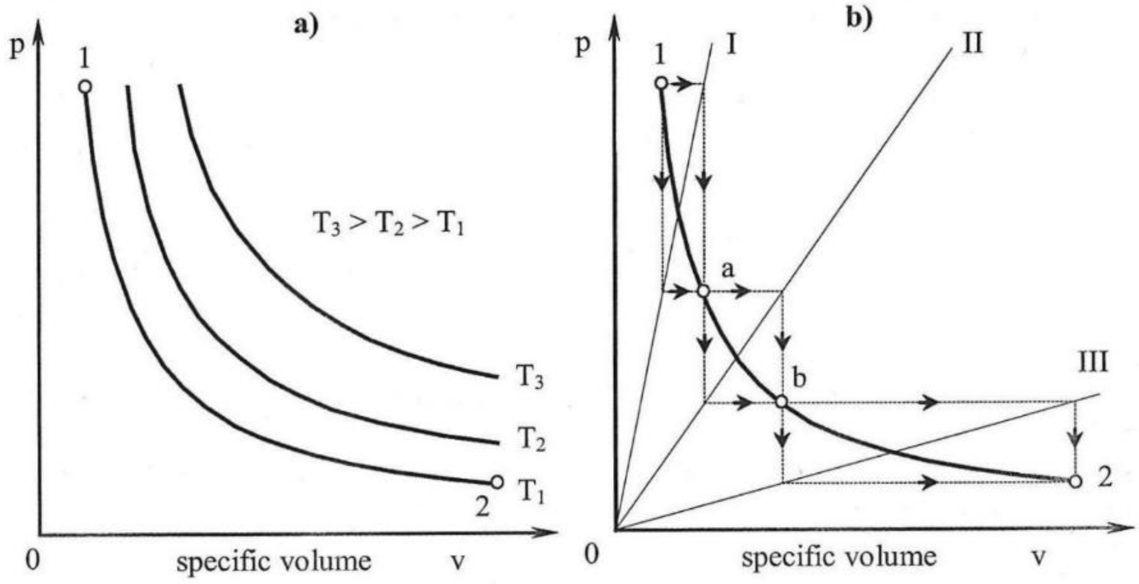


Fig. 6.1. Isothermal processes in the p–v coordinate system:
a) isotherms for various temperatures, b) geometrical construction of the isotherm

The equation of isotherm (6.1) is the equation of rectangular hyperbola (also called an equilateral hyperbola or a right hyperbola). In figure 6.1b is shown the example of one of geometrical constructions leading to the obtainment of the hyperbola, that is to say of the **isotherm**. First, in the *p–v* coordinate system, one ought to mark the initial point of the isothermal transformation (1), defined by original values of parameters of state; p_1 and v_1. Next one draws the radius (I) starting from the beginning of the coordinate system. Afterwards one draws lines parallel to the coordinate axis, which cross over through point (1) and marks their intersections with radius (I) and from these intersections draws next lines parallel to the coordinate axis. Their intersection marks the second point of the hyperbola, denoted with (*a*). We can repeat this procedure so many times, until we obtain such number of points which will permit to draw the curve with required accuracy.

→ Internal energy and enthalpy in the isothermal process

In compliance with the equations (4.32) and (4.34), because for $T = const$ $dT = 0$, the increase in the internal energy and the enthalpy in the isothermal transformation must be equal to 0:

$$du = c_v dT = 0 \tag{6.2a}$$

and:

$$di = c_p dT = 0 \tag{6.2b}$$

This means that both the value of the internal energy and the enthalpy are constant (and equal to the initial value) in progress of the process.

→ Heat and work in the isothermal process

The mechanical (expansion or compression) work of the process we can calculate using the equation (4.14), in which for p we introduce the expression obtained from equation (6.1a):

$$p = \frac{p_1 v_1}{v} \tag{a}$$

After introducing (a) into (4.14) we obtain:

$$l = \int_1^2 p \cdot dv = \int_1^2 \frac{p_1 v_1}{v} dv = p_1 v_1 \int_1^2 \frac{dv}{v} = p_1 v_1 \ln \frac{v_2}{v_1} \tag{6.3}$$

Using the equation (6.1b) and equation of state (6.1), we can write down the formula (6.3) in following form:

$$l = p_1 v_1\, ln \frac{p_1}{p_2} = RT_1\, ln \frac{p_1}{p_2} \tag{6.3a}$$

For the closed system, instead of the specific volume v one can use the total volume V, and then:

$$L = p_1 V_1 \ln \frac{p_1}{p_2} = MRT_1 \ln \frac{p_1}{p_2} = n(\mu R)T_1 \ln \frac{p_1}{p_2} \tag{6.4}$$

Determination of the heat and work at the isothermal process is not difficult, if noticed, that in compliance with the First Law of Thermodynamics:

$$dq_c = du + pdv = di - vdp$$

and because of $du = di = 0$, one can write:

$$dq_c = pdv = -\, vdp,$$

and finally:

$$q_c = l = l_t \tag{6.5}$$

or, for the closed system:

$$Q_c = L = L_t \tag{6.5a}$$

Because, in spite of heat supplied to the system, the temperature of gas (thermodynamic agent) is constant (the entire heat supplied is equal to performed work), from the formula (3.6) results, that (from the mathematical point of view) at the isothermal transformation the specific heat is infinitely large, heading (accordingly to the sign of heat q_c) for $+\infty$ or $-\infty$.

Isochoric Process

The **isochore** (the isochoric transformation, termed also the isometric process) we call the transformation occuring at the constant volume of a gas. From the equation of state of an ideal gas (the Clapeyron equation) results that for $V = const$:

$$v = \frac{RT}{p} = const \tag{6.6}$$

that is to say, for two different states of the gas:

$$\frac{p_1}{p_2} = \frac{T_1}{T_2} \tag{6.6a}$$

The formulas (6.6) and (6.6a) are the basic equations of the isochoric process, expressing also the so called Charles' law – see (2.4). The isochoric transformation in the p–v coordinate system is shown in figure 6.2.

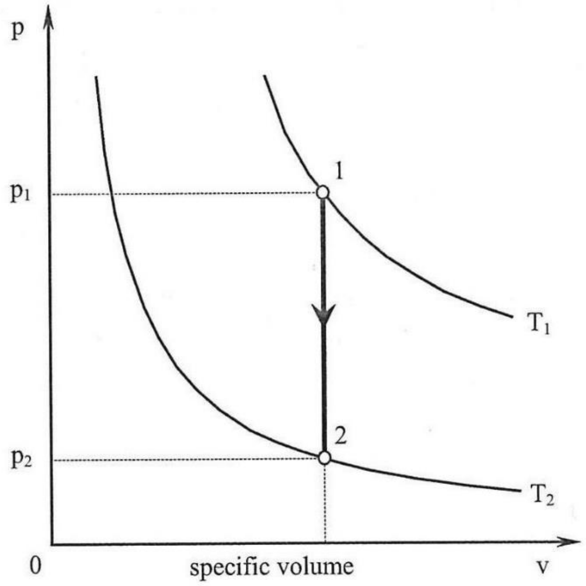


Fig. 6.2. Isochoric process in the p–v coordinate system between two isotherms: $T_1 > T_2$

→ **Internal energy and enthalpy in the isochoric process**

In compliance with equations (4.32) and (4.34), the increase in the internal energy and the enthalpy in the isochoric transformation amounts to:

$$du = c_v dT$$

and:

$$di = c_p dT$$

with reference to the finite quantities and for ideal gases:

$$\Delta u = u_2 - u_1 = c_v\left(T_2 - T_1\right) \tag{6.7}$$

$$\Delta i = i_2 - i_1 = c_v\left(T_2 - T_1\right) \tag{6.7a}$$

For closed systems, where in progress of the process the mass of gas in a system does not change:

$$\Delta U = U_2 - U_1 = Mc_v\left(T_2 - T_1\right) \tag{6.7b}$$

$$\Delta I = I_2 - I_1 = Mc_v\left(T_2 - T_1\right) \tag{6.7c}$$

If the transformation refers to a semiideal gas, instead of the constant value of the specific heat appears the mean specific heat and for the specific internal energy and the specific enthalpy, we obtain:

$$\Delta u = c_v\Big|_{t_1}^{t_2} \cdot (T_2 - T_1) \tag{6.8}$$

$$\Delta i = c_p\Big|_{t_1}^{t_2} \cdot (T_2 - T_1) \tag{6.8a}$$

Analogically one can modify the dependencies – equations (6.7) and (6.7a) – applied to the closed system.

→ **Heat and work in the isochoric process**

The **mechanical work** in the isochoric process is equal to 0 as we can see in figure 6.2 and what results at once from the definer formula (4.14) for $dv = 0$.

The **flow work** one can also calculate without difficulty from the definer dependency, for 1 kg:

$$l_t = -\int_1^2 v dp = -v(p_2 - p_1) = v(p_1 - p_2) = R(T_1 - T_2) \tag{6.9}$$

and, for a closed system:

$$L_t = -\int_1^2 V dp = -V(p_2 - p_1) = V(p_1 - p_2) = MR(T_1 - T_2) \tag{6.10}$$

The **heat in the isochoric transformation** one can appoint from the First Law of Thermodynamics (4.25) taking into account that $dv = 0$. So, for ideal gases:

$$dq_c = du = c_v dT \tag{6.11}$$

thus:

$$q_c = c_v(T_2 - T_1) \tag{6.12}$$

and for closed systems:

$$Q_c = Mc_v(T_2 - T_1) \tag{6.13}$$

For semiideal gases:

$$q_c = c_v\Big|_{t_1}^{t_2} \cdot (T_2 - T_1) \tag{6.14}$$

or for closed systems

$$Q_c = M \cdot c_v\Big|_{t_1}^{t_2} \cdot (T_2 - T_1) \tag{6.14a}$$

As it can be seen the entire heat in the isochoric transformation is used for the increase in the internal energy.

Isobaric Process

The **isobar** (the isobaric transformation) we call the transformation occuring at the constant pressure of a gas. From the equation of state of an ideal gas (the Clapeyron equation) results that for $p = const$:

$$p = \frac{RT}{v} = const \tag{6.15}$$

that is to say, for two different states of the gas:

$$\frac{v_1}{v_2} = \frac{T_1}{T_2} \tag{6.15a}$$

The formulas (6.15) and (6.15a), are the basic equations of the isobaric transformation, expressing also the so called Gay-Lussac's law – see chapter 2, equations (2.5) and (2.5a).

The isobaric transformation in the p–v coordinate system is shown in figure 6.3.

→ **Internal energy and enthalpy in the isobaric process**

In the isobaric transformation the internal energy and the enthalpy are expressed by the same dependencies as in the isochoric process (6.7) – (6.8).

→ **Heat and work in the isobaric process**

The **flow work** in the isobaric process is equal to 0 as we can see in figure 6.3 and what results at once from the formula (4.21c) for $dp = 0$.

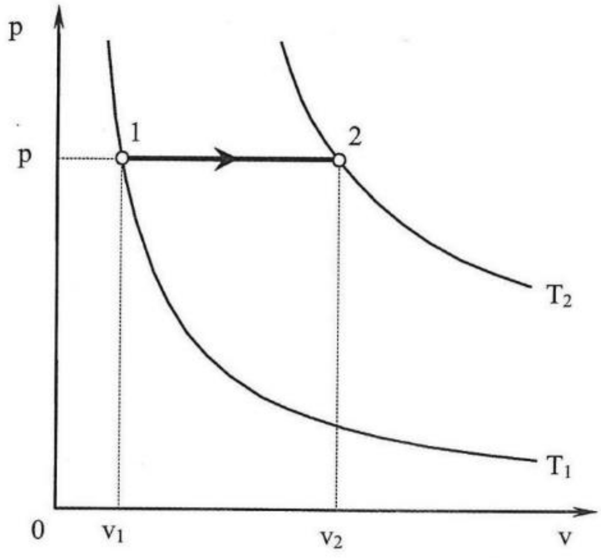


Fig. 6.3. Isobaric process in the p–v coordinate system, between two isotherms: $T_1 < T_2$

The **mechanical work** one can also calculate without difficulty from the definer dependency, for 1 kg of gas:

$$l = \int_1^2 p dv = p(v_2 - v_1) = R(T_2 - T_1) \tag{6.16}$$

and, for the closed systems (the entire mass of gas):

$$L = \int_1^2 V dp = p(V_2 - V_1) = MR(T_2 - T_1) \tag{6.17}$$

The **heat in the isobaric transformation** one can appoint from the First Law of Thermodynamics (4.26) taking into account that $dp = 0$. So, for ideal gases:

$$dq_c = di = c_p dT \tag{6.18}$$

thus:

$$q_c = c_p (T_2 - T_1) \tag{6.19}$$

and for closed systems:

$$Q_c = M c_p (T_2 - T_1) \tag{6.20}$$

For semiideal gases:

$$q_c = c_p \Big|_{t_1}^{t_2} \cdot (T_2 - T_1) \tag{6.21}$$

or for closed systems:

$$Q_c = M \cdot c_p\Big|_{t_1}^{t_2} \cdot (T_2 - T_1) \tag{6.21a}$$

As it can be seen the entire heat in the isobaric transformation is used for the increase in the enthalpy.

Isentropic Process – Reversible Adiabatic Process

The adiabate we call the transformation in which heat is neither transferred from the surroundings nor given back to it (there is no heat exchange with surroundings), that is to say $dq = 0$. It can occur if the process is so quick that there is no time for the heat flow (theoretically infinitely quickly), or if the system is absolutely insulated from its surroundings. In this case the work may be performed only at the expense of the internal energy of gas (thermodynamic agent). If in addition in the adiabatic transformation the friction does not appear, that is to say $dq_f = 0$, then such a process we call a **reversible adiabate or an isentrope**.

So, for the isentropic process:

$$dq + dq_f = dq_c = 0 \tag{6.22}$$

One ought to notice here that the fulfilment of the equation (6.22) does not always mean that simultaneously both components must be equal to 0. Because such a case may appear at which $dq_f > 0$ and $dq < 0$ and in sum $dq_c = 0$, such a transformation is not an adiabate.

A parameter of state which in the isentropic transformation keeps its constant value is the entropy $s = const$ that is to say $ds = 0$. Besides, from this parameter originates the name of the process. The definition of the entropy and its detailed discussion are included in the next chapter. Because in the isentropic process, despite $dq_c = 0$, the temperature of the thermodynamic agent (gas) changes, from the formula 3.6 results that (from the mathematical point of view) in this process the specific heat is equal to 0.

The **equation of isentropic process** we can obtain using the equations of the I Law of Thermodynamics in forms (4.25) and (4.26) into which we put $dq_c = 0$. Then we will obtain:

$$du + pdv = 0$$

and:

$$di - vdp = 0$$

if now we take into account the formulas for internal energy (4.32) and for enthalpy (4.34), we will obtain:

$$c_v dT = -pdv \tag{a}$$

and:

$$c_p dT = vdp \tag{b}$$

if now we divide the equation (b) by (a), appropriately allocate the variables and notice that $c_p/c_v = \kappa$, then we will obtain one of the forms of the differential equation of isentrope:

$$\kappa\frac{dv}{v}+\frac{dp}{p}=0 \tag{6.23}$$

→ **Equation of isentropic process of ideal gases**

For an ideal gas the exponent of isentrope is a constant value, $\kappa = const.$ Thereby the equation (6.23) one can integrate:

$$\kappa\int_{v_1}^{v_2}\frac{dv}{v}=-\int_{p_1}^{p_2}\frac{dp}{p}$$

obtaining:

$$\kappa\cdot\ln\frac{v_2}{v_1}=\ln\frac{p_1}{p_2}$$

If we use now the laws of logarithmic calculus we will at last obtain the **equation of the isentropic process** for finite quantities**,** termed the **Poisson's equation**:

$$\left(\frac{v_2}{v_1}\right)^{\kappa}=\frac{p_1}{p_2} \tag{6.24}$$

or:

$$p_1 v_1^{\kappa}=p_2 v_2^{\kappa} \tag{6.24a}$$

Of course in the equation (6.24) one can use other pairs of thermodynamic parameters, eliminating the undesirable parameters by means of the Clapeyron equation, e.g.:

- if in the equation (6.24) we eliminate pressures using the equation of state $p = RT/v$, after the transformation we will obtain another form of the Poisson's equation:

$$\left(\frac{v_2}{v_1}\right)^{\kappa-1}=\frac{T_1}{T_2}$$

or:

$$\frac{v_2}{v_1}=\left(\frac{T_1}{T_2}\right)^{\frac{1}{\kappa-1}} \tag{6.24b}$$

– if in the equation (6.24) we eliminate the specific volume using the equation of state $v = RT/p$, after the transformation we will obtain another form of the Poisson's equation:

$$\left(\frac{T_1}{T_2}\right)^{\frac{\kappa}{\kappa-1}} = \frac{p_1}{p_2}$$

or:

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{\kappa-1}{\kappa}} \tag{6.24c}$$

The isentropic process in the p–v coordinate system (between two points located on isotherms) is shown in figure 6.4.

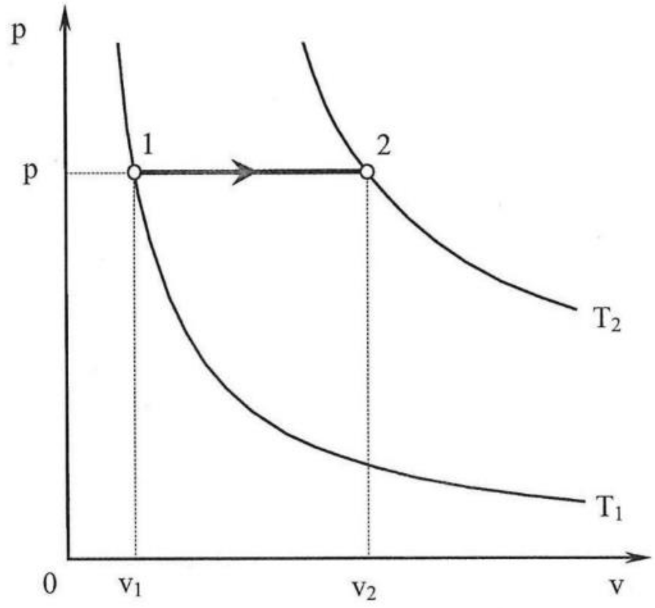


Fig. 6.4. Isentropic process of an ideal gas in the p–v coordinate system, between two isotherms; $T_1 > T_2$

→ Internal energy and enthalpy in the isentropic process

In the isentropic process the internal energy and the enthalpy are expressed by the same dependencies as in the isochoric transformation – equation (6.7) – (6.8).

→ Work in the isentropic process

The **mechanical work** we can calculate by integrating the definitional dependency of the work for 1 kg of gas ($dl = pdv$) in which we eliminate the pressure p by means of the equation of transformation $pv^{\kappa} = p_1 v_1^{\kappa}$. However, we can also derive the expression of the mechanical (expansion or compression) work directly from the equation of the First Law of Thermodynamics.

Below the second manner is shown. Because $dq_c = du + dl$ but $dq_c = 0$, we can write $dl = -du$ and after the integration and the use of the equation (3.13), we obtain:

$$l = u_1 - u_2 = c_v(T_1 - T_2) = \frac{1}{\kappa - 1} R(T_1 - T_2) \tag{6.25}$$

If now we use the Clapeyron equation to eliminate the temperatures and then we use the Poisson's equation of the isentrope, we can obtain from the dependency (6.25) the following forms of formulas of the mechanical work:

$$l = \frac{1}{\kappa - 1}(p_1 v_1 - p_2 v_2) = \frac{1}{\kappa - 1} R T_1 \left(1 - \frac{T_2}{T_1}\right) = \frac{1}{\kappa - 1} R T_1 \left[1 - \left(\frac{v_1}{v_2}\right)^{\kappa - 1}\right] =$$

$$= \frac{1}{\kappa - 1} p_1 v_1 \left[1 - \left(\frac{v_1}{v_2}\right)^{\kappa - 1}\right] = \frac{1}{\kappa - 1} p_1 v_1 \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\kappa - 1}{\kappa}}\right] \tag{6.25a}$$

Of course for a closed system (with the entire mass of gas *M*):

$$L = M \cdot l \tag{6.25b}$$

or else instead of the specific volume, into the expression (6.25a) we put total volume *V*.

The **flow work** in the isentropic transformation we can determine at once from the second form of the First Law of Thermodynamics $dq_c = di + dl_t$ from which for $dq_c = 0$ results that $dl_t = -di$, that is to say:

$$l_t = i_1 - i_2 = c_p(T_1 - T_2) = \kappa \cdot c_v(T_1 - T_2) = \kappa \cdot l \tag{6.26}$$

So the flow work is κ times greater than the mechanical work (what results from the equation 3.12) and we obtain any form of the expression of the flow work by multiplying the selected form of the dependency (6.25) or (6.25a) by the isentropic exponent κ.

For closed systems, of course, in force is the dependency:

$$L_t = \kappa \cdot L \tag{6.27}$$

→ Isentropic process for semiideal gases

For semiideal gases the specific heats are functions of the temperature $c_v = f_1(T)$, $c_p = f_2(T)$ so consequently, also the exponent of the isentrope must be a function of the temperature, $\kappa = f_3(T)$. Only in the special case of $f_2(T) = f_1(T)$ the exponent κ will be constant. The isentrope equation for semiideal gases one can obtain from the differential equation of the isentrope, however it is more convenient to use the equation for another pair of parameters of state than the expression (6.23).

After the substitution in the equation (4.25) of $dq_c = 0$ and $du = c_v dT$, we obtain: $c_v dT + pdv = 0$. If now we eliminate the pressure using the Clapeyron equation ($p = RT/v$) and we divide its both sides by T, we will at last obtain the second form of the differential equation of the isentrope:

$$c_v \frac{dT}{T} + R\frac{dv}{v} = 0 \tag{6.28}$$

The integration of this equation within the range from conventionally accepted parameters of the datum level (T_0, v_0) to their actual value (T, v) leads to following function:

$$\int_{T_0}^{T_1} c_v \frac{dT}{T} + R\ln\frac{v}{v_0} = 0 \tag{6.29}$$

in which the integral segment being exclusively a function of the temperature is called the **Nusselt's function.**

After determination of the value of the Nusselt's function (by numerical methods or from tables), this equation allows to calculate the volumes ratio, if the extreme temperatures of the transformation are known or to calculate the final temperature if the initial temperature and the ratio of the volumes are known. The equation (6.29) may be also transformed to the form in which the volumes ratio one replaces with the suitable function of the pressures ratio and the temperatures ratio. Nevertheless, as it can be seen, it is not possible to develop an easy to use function in a form similar to the expression (6.24).

On that account, unless one foresees application of the numerical methods, for rough estimates one can use expressions given for the isentrope of the ideal gas, however in place of the constant value of the isentrope exponent κ, one puts in the average value κ calculated for the temperature range of the given process (e.g. using the dependence 3.21).

Polytropic Process

The **polytrope** with reference to the thermodynamic transformations one can understand in two ways. The basic meaning of the word "polytrope" is descended from the Greek "politropos" – manifold, however in a free translation, referred to the thermodynamics it is understood as the plurality of the paths (of the processes). In this meaning the polytrope is a transformation embracing the whole family of various transformations (processes), also including the characteristic transformations shown above and – as it will be shown further – such an interpretation is absolutely correct.

The second meaning of the "polytrope" (or of the polytropic process) – more often used in thermodynamics – has a narrower denotation than the first one. In this understanding the polytrope we call the transformation defined by the equation:

$$pv^m = const \tag{6.30}$$

where:

p – pressure of gas, in [Pa],
v – specific volume, in [m^3/kg],
m – a constant called the polytropic exponent (or polytropic index),

in which m = *constant* but has the other value than the exponents of other characteristic transformations. Some textbooks call such the curve an ideal polytrope.

The polytropic exponent m one can interpret as the dimensionless specific heat, on that Zeuner based his definition of the polytropic transformation. This interpretation is based on the reasoning, wherein it is assumed that the energy transfer takes place simultaneously after the manner of heat and work. So, the equations of the First Law of Thermodynamics (4.25) and (4.26), taking into account the expression (3.5), we can write down as follows:

$$dq_c = cdT = c_v dT + pdv \tag{a}$$

and:

$$dq_c = cdT = c_p dT - vdp \tag{b}$$

these equations we can also write down in the forms:

$$(c - c_v)dT = pdv \tag{a1}$$

and:

$$(c - c_p)dT = -vdp \tag{b1}$$

if now we divide equation (b1) by (a1) and appropriately allocate the variables, then we will obtain the definitional expression of the polytropic exponent:

$$m = \frac{c - c_p}{c - c_v} = -\frac{vdp}{pdv} = \frac{d(\ln p)}{d(\ln v)} \tag{6.31}$$

Using the equation (6.30) we can write down the equation of the polytropic transformation occurring between two states, for any gas (not only of an ideal gas):

$$p_1 v_1^{\,m} = p_2 v_2^{\,m} \tag{6.32}$$

or, after the transformation:

$$\left(\frac{v_2}{v_1}\right)^m = \frac{p_1}{p_2} \tag{6.32a}$$

The polytrope equation (6.32a) one can also obtain directly, integrating the transformed form of equation (6.31) between states 1 and 2:

$$m\frac{dv}{v} + \frac{dp}{p} = 0 \tag{6.33}$$

obtaining:

$$m \cdot \ln\frac{v_2}{v_1} + \ln\frac{p_1}{p_2} = 0 \tag{6.33a}$$

what directly leads to equations (6.32) and (6.32a).

→ Polytropic process of an ideal gas

For ideal gases the equations (6.32) and (6.32a) we can transform using the Clapeyron equation, obtaining dependencies in which appears also the temperature.

- if in equation (6.32a) we eliminate the pressures by means of the equation of state: $p = RT/v$, then after this transformation we will obtain another form of this equation:

$$\left(\frac{v_2}{v_1}\right)^{m-1} = \frac{T_1}{T_2}$$

or:

$$\frac{v_2}{v_1} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{m-1}} \tag{6.34}$$

- instead, if in the equation (6.32a) we eliminate the specific volumes by means of the equation of state: $v = RT/p$, then after this transformation we will obtain the next form of the equation:

$$\left(\frac{T_1}{T_2}\right)^{\frac{m}{m-1}} = \frac{p_1}{p_2}$$

or:

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{m-1}{m}} \tag{6.35}$$

→ Work and heat in the polytropic process of ideal gases

We can calculate the specific heat in the polytropic transformation at the known exponent $m = const$, transforming the definitional expression (6.31):

$$c = c_v \frac{m-\kappa}{m-1} = c_p \frac{m-\kappa}{\kappa(m-1)} \tag{6.36}$$

The **mechanical work** we can calculate integrating its definitional dependency for 1 kg of gas ($dl = pdv$), so:

$$l = \int_1^2 pdv$$

If now we eliminate the pressure (occurrent under the integral) by means of the equation of the polytropic process $pv^m = p_1v_1^m$:

$$p = p_1 v_1^m \frac{1}{v^m}$$

we will obtain:

$$\begin{aligned} l &= p_1 v_1^m \int_1^2 \frac{1}{v^m} dv = -\frac{1}{m-1} p_1 v_1^m \frac{1}{v^{m-1}} \Bigg|_1^2 = \\ &= -\frac{1}{m-1} p_1 v_1 \left(\frac{v_1}{v}\right)^{m-1} \Bigg|_1^2 = \\ &= -\frac{1}{m-1} p_1 v_1 \left[\left(\frac{v_1}{v_2}\right)^{m-1} - \left(\frac{v_1}{v_1}\right)^{m-1}\right] = \\ &= \frac{1}{m-1} p_1 v_1 \left[1 - \left(\frac{v_1}{v_2}\right)^{m-1}\right] \end{aligned} \tag{6.37}$$

Applying the expression (6.37) one can obtain other formulas for this work, using the equations of the polytropic transformation (6.34) and (6.35):

$$l = \frac{1}{m-1} p_1 v_1 \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{m-1}{m}}\right] = \frac{1}{m-1} p_1 v_1 \left[1 - \frac{T_2}{T_1}\right] \tag{6.37a}$$

Another interesting form of the equation (6.37a) may be obtained by replacing (according to the Clapeyron equation) the product $p_1 v_1$ with RT_1.

$$l = \frac{1}{m-1} RT_1 \left(1 - \frac{T_2}{T_1}\right) = \frac{1}{m-1} R(T_1 - T_2) = \frac{1}{m-1}(p_1 v_1 - p_2 v_2) \tag{6.37b}$$

We can also calculate the **flow work** by integrating its definitional dependency ($dl_t = -vdp$) but considerably more quickly we may obtain the suitable dependency, if we notice that using the transformed form of the differential polytrope equation (6.33).

$$m\frac{dv}{v} = -\frac{dp}{p}$$

after multiplying both sides of the equation by the product pv, we obtain:

$$mpdv = -vdp$$

or, in other words:

$$mdl = dl_t$$

So, also for finite values we can write:

$$l_t = ml \tag{6.38}$$

and accordingly, for a closed system (for the entire mass of gas):

$$L_t = mL \tag{6.39}$$

Thus, in the polytropic process the flow work is m times greater than the mechanical work. The **internal energy** and the **enthalpy** one can calculate using the dependencies (6.7) and (6.7a).

The heat of the polytropic process one can calculate as follows:

$$q_c = \int_1^2 c dT = c(T_2 - T_1) = c_v \frac{m-\kappa}{m-1}(T_2 - T_1) \tag{6.40}$$

For semiideal gases, the heat of the polytropic process may be calculated using another formula (with the average specific heat in place of the constant specific heat):

$$q_c = \int_1^2 c dT = c\Big|_{T_1}^{T_2} (T_2 - T_1) \tag{6.41}$$

→ **Polytrope as a transformation (process) representing family of transformations (processes)**

At the beginning of this subsection we said that the polytrope is a transformation including the whole family of various processes, therein also the characteristic transformations discussed previously, which are only the special cases of the polytrope. One can notice that the equation of polytrope $pv^m = const$ will assume various forms, depending on the value of the exponent m.

So, for the exponent:

a) $\boldsymbol{m = 1}$

The polytrope equation transforms itself into the isotherm equation
($pv = const$ or $T = const$ and $dT = 0$) for which the specific heat of the process tends to the infinity, $c \rightarrow \pm\infty$.

b) $\boldsymbol{m = 0}$

The polytrope equation transforms itself into the isobar equation
($T/v = const$ or $p = const$ and $dp = 0$), for which the specific heat of the transformation is equal to c_p; ie. $c = c_p$.

c) $\boldsymbol{m \rightarrow \infty}$

The polytrope equation transforms itself into the isochore equation
($T/p = const$ or $v = const$ and $dv = 0$). It can be seen after the transformation of the equation of polytrope to the form:

$$\left(\frac{p_1}{p_2}\right)^{\frac{1}{m}} = \frac{v_2}{v_1} \qquad \text{from which results } v_2 = v_1 \text{ for } 1/m = 0$$

The specific heat of this process is equal to c_v; i.e. $c = c_v$.

d) **m = κ**

the polytrope equation transforms itself into the isentrope equation (ie. reversible adiabate) for which $s = const$ ($ds = 0$ and $dq_c = 0$) and the specific heat of the transformation is equal to 0; ie. $c = 0$.

However, it is worth to notice here, that in case of $dq = 0$ but $dq_f > 0$ (irreversible adiabate) also $dq_c > 0$ and then both the transformation exponent $m \neq \kappa$ and its specific heat $c \neq 0$.

The family of curves representing all of the characteristic transformations mentioned above in the *p–v* coordinate system is shown in figure 6.5.

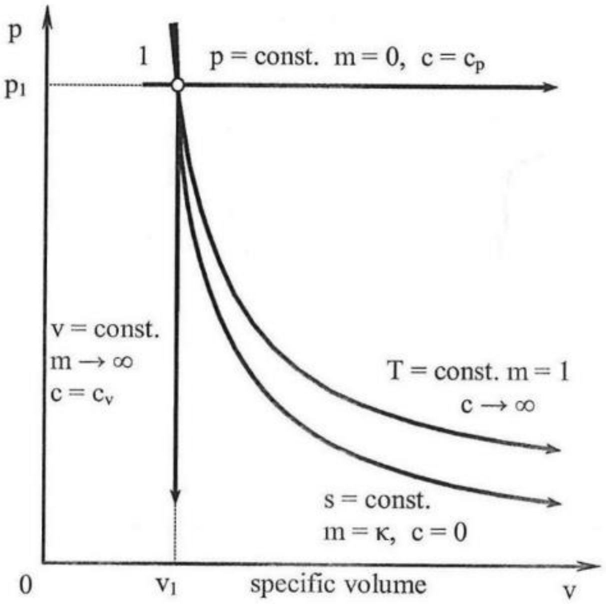


Fig. 6.5. Family of characteristic transformations resultant from the polytrope equation at various values of the exponent of polytrope (m), in the *p–v* coordinate system

In engineering thermodynamics, practically of the greatest importance is the area of transformations contained in figure 6.5 between the isotherm and the isentrope, this means at exponent: $\kappa > m > 1$.

The character of the politrope equation $pv^m = const$ (and therefore also of all characteristic transformations) cause that in the logarithmic coordinate system these curves straighten. It is easy to demonstrate, because after finding the logarithm of both sides of the politrope equation (6.30), we obtain:

$$\lg\left(pv^m\right) = \lg(const)$$

or:

$$\lg p = -m \lg v + const \tag{6.42}$$

As we can see, the equation (6.42) determines the linear function in the *lg p–lg v* coordinate system. Thus the polytrope equation – and each curve of the (6.30) form – straightens in the logarithmic coordinate system. It is intelligible, if we notice that the logarithmic function and the exponential function are the inverses of each other.

The family of curves representing all of the above–mentioned characteristic transformations (processes) in the logarithmic coordinate system (*lg p–lg v*) is shown in figure 6.6.

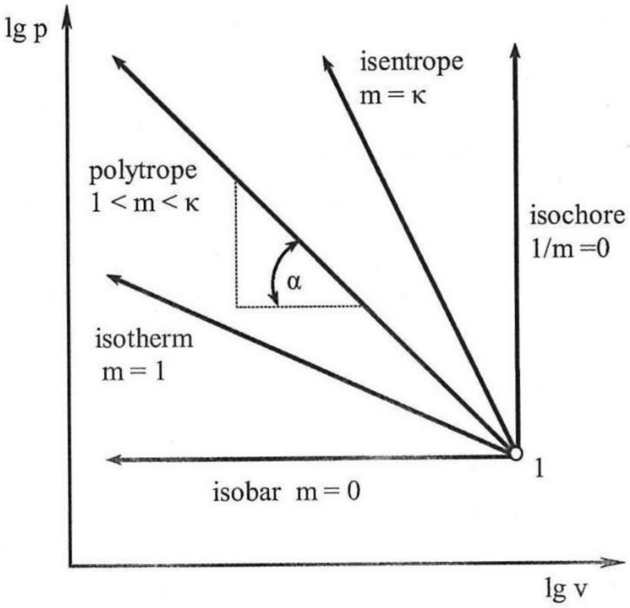


Fig. 6.6. Family of characteristic transformations resultant from the polytrope equation at various values of the exponent m, in the lg *p*–lg *v* coordinate system

The tangent of the angle inclinations (α) of the straight line being a transformed curve in the logarithmic coordinate system is equal to the exponent of equation of this process:

$$m = tg\alpha = -\frac{d(\lg p)}{d(\lg v)} \tag{6.43}$$

The effect of the straightening of curves of the (6.30) form in the logarithmic coordinate system one can use to:

- Check whether the curve of the process in the *p–v* coordinate system (e.g. experimentally appointed) is a polytrope. If it is a polytrope its graph will straighten in the logarithmic coordinate system (lg *p* – lg *v*).
- Qualify the exponent of the process equation, if one can assume that the process fulfils the equation (6.30), using the effect of its straightening in the logarithmic coordinate system (lg *p* – lg *v*). One can make this graphically on the graph, calculating *tg α*, or else directly by finding the logarithm of the equation of the process (6.32) for two points:

$$m = \frac{\lg \frac{p_2}{p_1}}{\lg \frac{v_1}{v_2}} \tag{6.44}$$

Exercise Procedure:

1. Aim of the exercise

Determining the polytropic exponent of a thermodynamic process of compression.

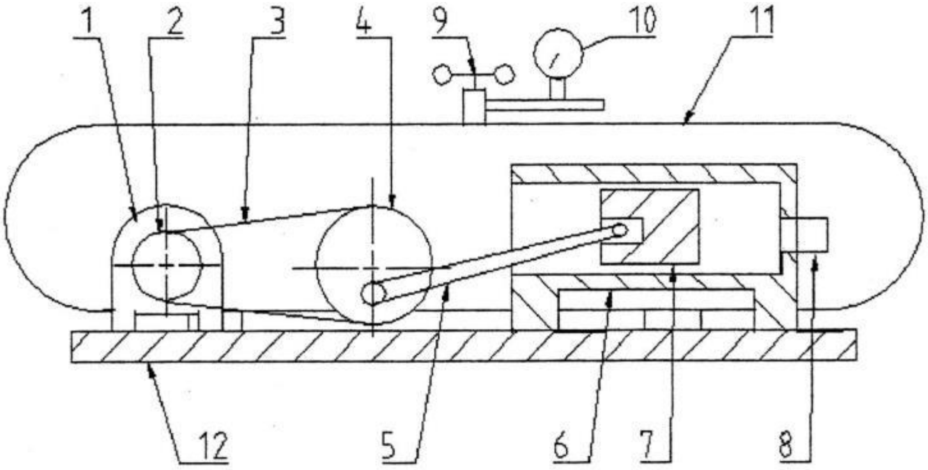
2. Procedure of the exercise

1. Registration of compressor cylinder pressure variation (indication chart)
2. Determination of the value of the polytropic exponent, the actual compression curve by various methods
3. Comparison of the cycle, actual compression curve with ideal compression; isothermal and isentropic

3. The course of the exercise

Determine the polytropic exponent of the actual compression process by two methods, based on the indication chart.

Scheme of the exercise stand:



Description of the compressor schematic:
1. AC electric motor.
2. small pulley attached to the shaft of the electric motor.
3. drive V-belt.
4. large pulley coupled to the slider-crank mechanism.
5. the “crank.”
6. piston housing.
7. Compression system piston.
8. Duct connecting the piston chamber (combustion chamber) to the compressed medium tank.
9. discharge valve.
10. compressed air pressure gauge.
11. compressed air tank.

In point No. 8. a mechanical indicator is mounted, which records the course of pressure as a function of piston stroke in the compressor cylinder.

4. Report card

1. Based on the provided graph use two methods to calculate the polytropic exponent
2. Compare the obtained polytrope (for average m) plotted with the isotherm (m=1) and isentrope (m = 1.4) process.
3. Calculate the temperature and density of air in pt. 2 of the polytropic process.
4. Calculate the flow work, expansion work, and absolute work and heat transformation for polytropic, isotherm, and isentropic adiabate.
5. Calculate work flux (power) for the assumed compressor number of strokes n = 130 rpm
6. Discussions and Conclusions

4.1. Polytropic indes calculation

1. Method 1

Calculation of the polytropic exponent by dividing the polytropic compression process of air (section 1 - 2 from the indicator diagram of the reciprocating compressor) into 10 sections and reading the values of volume V and pressure p for each of the resulting points. The values of pressure and volume are logarithmized and the partial values are inserted into the formula and determine the polytropic exponent.

$$\mathbf{m_i = \frac{\ln \frac{p_2}{p_1}}{\ln \frac{V_1}{V_2}}}$$

Results:

L. p.	V [m³]	p [Pa]	ln (V) [m³]	ln (p) [Pa]	**m**
1					
				M_{avg}	

1. Method 2

Calculation of the polytropic exponent using the known parameters of the air gas state of the initial 1 and final 2 transformation points from the indicator chart. The m_i values found in the measurement table were calculated using the formula:

The m_i values found in the measurement table were calculated based on the formula::

$$p \cdot V^m = idem \quad \Longrightarrow \quad p_1 \cdot V_1^m = p_2 \cdot V_2^m$$

$$\left(\frac{V_1}{V_2}\right)^m = \frac{p_2}{p_1} \quad \Longrightarrow \quad m \cdot \ln\frac{V_1}{V_2} = \ln\frac{p_2}{p_1}$$

$$\mathbf{m_i = \frac{\ln \frac{p_2}{p_1}}{\ln \frac{V_1}{V_2}}}$$

The value of m_{avg} was calculated using the values of m_i from method 2. And 3. As the arithmetic average of the two methods:

$$\mathbf{m_{avg} = \frac{\sum m_i}{i}}$$

5. Calculations

5.1. Compare the obtained polytrope (for average m) plotted with the isotherm (m=1) and isentrope (m = 1.4) process.

The averaged polytrope is to be compared with ideal process on a graph **ln p = f (ln V)**:

- **Isotherm,** index m = 1
- **Izentrope,** index m = κ = 1,4 (air)

5.2. Calculate the temperature and density of air in pt. 2 of the polytropic process

Air temperaturę can be obtained with the equation:

$$\mathbf{T_2 = T_1 \cdot \left(\frac{p_2}{p_1}\right)^{\frac{m_{avg}-1}{m_{avg}}} \qquad [K]}$$

where: T_1 = 20°C = 293.15 K

p_1, p_2, m_{avg} – pressure vallues for pt. 1 and 2 are to be read from previously mentioned table, and should be assumed for isentropic process as $m_{avg} = \kappa = 1{,}4$

Air density should be calculated as :

$$\mathbf{\rho_2 = \frac{p_2}{R \cdot T_2} \qquad [\frac{kg}{m^3}]}$$

where: R = 286,7 [J/kg · K] – individual gas constant for air

$p_{2,} T_2$ – from previous equations

5.3. Calculate the flow work, expansion work, and absolute work and heat transformation for polytropic, isotherm, and isentropic adiabate.

Politrope

- Flow work $\mathbf{L_{t\,1-2} = \frac{m_{śr}}{m_{śr}-1} \cdot RT_1 \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{m_{śr}-1}{m_{śr}}}\right] \quad \left[\frac{J}{kg}\right]}$
- Expansion work $\mathbf{L_{1-2} = \frac{L_{t\,1-2}}{m_{śr}} \quad \left[\frac{J}{kg}\right]}$
- Process heat $\mathbf{q_{1-2} = c \cdot \Delta T = c_v \cdot \frac{m_{śr}-\kappa}{m_{śr}-1}(T_2 - T_1) \quad \left[\frac{J}{kg}\right]}$

where: c_v – specific heat at constant volume

$$\mathbf{c_v = \frac{\kappa}{\kappa - 1} \cdot R \qquad \left[\frac{J}{kg \cdot K}\right]}$$

Isoterma

Flow work, expansion work, process heat ,

$$\mathbf{L_{t\,1-2} = L_{1-2} = q_{1-2} = R \cdot T \cdot ln\frac{V_2}{V_1} = R \cdot T \cdot ln\frac{p_1}{p_2}} \quad \left[\frac{J}{kg}\right]$$

where: $T = T_1 = T_2 = 293{,}15$ K

R = 286,7 [J/kg · K] – individual gas constant of air

Izentropa

Flow work

$$\mathbf{L_{t\,1-2} = c_p \cdot (T_1 - T_2) = \frac{\kappa}{\kappa - 1} \cdot R \cdot (T_1 - T_2)}$$

where: c_p – specific heat at constant pressure

Expansion work $\mathbf{L_{1-2} = \frac{L_{t\,1-2}}{\kappa}}$ $\left[\frac{J}{kg}\right]$

Process heat $\mathbf{q_{1-2} = 0}$

5.4. <u>Calculate flow work flux (power) for assumed numer of rotations (strokes) of the compressor n = 130 [obr./min]</u>

Mass flow $\mathbf{\dot{M} = \dot{V}_{sk} \cdot \eta \cdot \rho_1 \cdot n}$ $\left[\frac{kg}{s}\right]$

where: n – rotations per second

$$\mathbf{130\ \left[\frac{rot}{min}\right] = 130 \cdot \frac{1}{60}\left[\frac{rot}{s}\right] = 2{,}17\left[\frac{rot}{s}\right]}$$

$$\mathbf{\eta = 80\% = 0{,}8}$$

$\dot{V}_{sk} = \dot{V}_k - \dot{V}_p$ – read from the indicator graph [m^3]

Volume flow rate from the beggining $\dot{V}_p$ to the end $\dot{V}_k$ of the graph

ρ_1 – density at initial conditions $\mathbf{\rho_1 = \frac{p_1}{R \cdot T_1}}$ $\left[\frac{kg}{m^3}\right]$

Flow work flux /compressor power/

$$\mathbf{\dot{L_t} = \dot{M} \cdot |L_{t\,1-2}|} \left[\frac{J}{s} = W\right]$$

$\mathbf{L_{t\,1-2}}$ **–** flow work of the polytropic transformation

<u>6) Conclusions and final remarks</u>

[1] References

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2. Moran M., Shapiro H., Fundamentals of engineering thermodynamics, John Wiley & Sons, 2nd ed., New York, 1992
3. Çengel Y., Boles M., Thermodynamics : an engineering approach, McGraw-Hill, 7th ed., New York, 2011
4. Kestin J.: Course in Thermodynamics, vol. 1,2 Hemisphere Publishing Company, New York 1979.

[2] Sample questions

1. Equation of state of ideal gas for isochoric, isothermal, isobaric, isentropic and polytropic process
2. Methods of determining he polytropic index m
3. Process heat, absolute (mechanical) work and technical (flow) work in perfect gas processes.
4. Diagrams of characteristic processes of the polytropic equation at different values of the index m