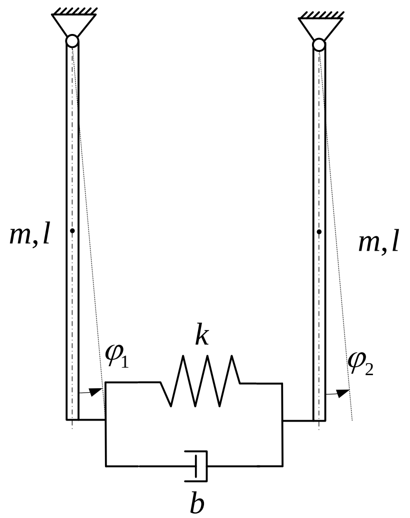
TEMAT:	**Drgania swobodne tłumione o wielu stopniach swobody**



$$E=\frac{1}{2}\frac{1}{3}ml^2\left(\dot{\varphi}_1^2+\dot{\varphi}_2^2\right)$$

$$V=-mg\frac{l}{2}\left(\cos\varphi_1+\cos\varphi_2\right)+\frac{1}{2}kl^2\left(\varphi_1-\varphi_2\right)^2$$

$$D=\frac{1}{2}bl^2\left(\dot{\varphi}_1-\dot{\varphi}_2\right)^2$$

$$\frac{1}{3}ml^2\ddot{\varphi}_1+\frac{mgl}{2}\varphi_1+bl^2\left(\dot{\varphi}_1-\dot{\varphi}_2\right)+kl^2\left(\varphi_1-\varphi_2\right)=0$$

$$\frac{1}{3}ml^2\ddot{\varphi}_2+\frac{mgl}{2}\varphi_2-bl^2\left(\dot{\varphi}_1-\dot{\varphi}_2\right)-kl^2\left(\varphi_1-\varphi_2\right)=0$$

$$\varphi_1=A_1e^{rt}\qquad \dot{\varphi}_1=A_1re^{rt}\qquad \ddot{\varphi}_1=A_1r^2e^{rt}$$

$$\varphi_2=A_2e^{rt}\qquad \dot{\varphi}_2=A_2re^{rt}\qquad \ddot{\varphi}_2=A_2r^2e^{rt}$$

gdzie: $r=i\omega$

$$M\ddot{\varphi}+B\dot{\varphi}+K\varphi=0$$

$$\begin{bmatrix}\frac{1}{3}ml^2 & 0\\ 0 & \frac{1}{3}ml^2\end{bmatrix}\begin{bmatrix}\ddot{\varphi}_1\\ \ddot{\varphi}_2\end{bmatrix}+\begin{bmatrix}bl^2 & -bl^2\\ -bl^2 & bl^2\end{bmatrix}\begin{bmatrix}\dot{\varphi}_1\\ \dot{\varphi}_2\end{bmatrix}+\begin{bmatrix}\frac{mgl}{2}+kl^2 & -kl^2\\ -kl^2 & \frac{mgl}{2}+kl^2\end{bmatrix}\begin{bmatrix}\varphi_1\\ \varphi_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$r^2\begin{bmatrix}\frac{1}{3}ml^2 & 0\\ 0 & \frac{1}{3}ml^2\end{bmatrix}\begin{bmatrix}A_1\\ A_2\end{bmatrix}+r\begin{bmatrix}bl^2 & -bl^2\\ -bl^2 & bl^2\end{bmatrix}\begin{bmatrix}A_1\\ A_2\end{bmatrix}+\begin{bmatrix}\frac{mgl}{2}+kl^2 & -kl^2\\ -kl^2 & \frac{mgl}{2}+kl^2\end{bmatrix}\begin{bmatrix}A_1\\ A_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}\frac{1}{3}ml^2r^2+bl^2r+\frac{mgl}{2}+kl^2 & -\left(kl^2+bl^2r\right)\\ -\left(kl^2+bl^2r\right) & \frac{1}{3}ml^2r^2+bl^2r+\frac{mgl}{2}+kl^2\end{bmatrix}\begin{bmatrix}A_1\\ A_2\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$W=\begin{vmatrix}\frac{1}{3}ml^2r^2+bl^2r+\frac{mgl}{2}+kl^2 & -\left(kl^2+bl^2r\right)\\ -\left(kl^2+bl^2r\right) & \frac{1}{3}ml^2r^2+bl^2r+\frac{mgl}{2}+kl^2\end{vmatrix}=$$

$$=\left(\frac{1}{3}ml^2r^2+bl^2r+\frac{mgl}{2}+kl^2\right)^2-\left(kl^2+bl^2r\right)^2=0$$

$$\left|\frac{1}{3}ml^2r^2+bl^2r+\frac{mgl}{2}+kl^2\right|=\left|kl^2+bl^2r\right|$$

$$\frac{1}{3}ml^2r^2+bl^2r+\frac{mgl}{2}+kl^2=kl^2+bl^2r$$

$$\frac{1}{3}ml^2r^2+\frac{mgl}{2}=0$$

$$r^2=-\frac{3g}{2l}$$

$$r_1=i\sqrt{\frac{3g}{2l}}\quad r_2=-i\sqrt{\frac{3g}{2l}}$$

$$\frac{1}{3}ml^2r^2+bl^2r+\frac{mgl}{2}+kl^2=-kl^2-bl^2r$$

$$\frac{1}{3}ml^2r^2+2bl^2r+\left(\frac{mgl}{2}+2kl^2\right)=0$$

$$r^2+\frac{6b}{m}r+\left(\frac{3g}{2l}+\frac{6k}{m}\right)=0$$

$$\Delta=\frac{36b^2}{m^2}-4\left(\frac{3g}{2l}+\frac{6k}{m}\right)=4\left[\frac{9b^2}{m^2}-\left(\frac{3g}{2l}+\frac{6k}{m}\right)\right]$$

$$r_3=-\frac{3b}{m}+i\sqrt{\frac{3g}{2l}+\frac{6k}{m}-\frac{9b^2}{m^2}}$$

$$r_4=-\frac{3b}{m}-i\sqrt{\frac{3g}{2l}+\frac{6k}{m}-\frac{9b^2}{m^2}}$$

$$\omega_0=\sqrt{\frac{3g}{2l}}\qquad \omega_t=\sqrt{\frac{3g}{2l}+\frac{6k}{m}-\frac{9b^2}{m^2}}\qquad \beta=\frac{3b}{m}$$

$$x_{11}=A_{11}e^{r_1t}\quad x_{12}=A_{12}e^{r_2t}\quad x_{13}=A_{13}e^{r_3t}\quad x_{14}=A_{14}e^{r_4t}$$

$$x_{21}=A_{21}e^{r_1t}\quad x_{22}=A_{22}e^{r_2t}\quad x_{21}=A_{23}e^{r_3t}\quad x_{24}=A_{24}e^{r_4t}$$

$$x_{11}=A_{11}\cos\omega_0t\quad x_{12}=A_{12}\sin\omega_0t\quad x_{13}=A_{13}e^{-\beta t}\cos\omega_tt\quad x_{14}=A_{14}e^{-\beta t}\cos\omega_tt$$

$$x_{21}=A_{21}\cos\omega_0t\quad x_{22}=A_{22}\sin\omega_0t\quad x_{13}=A_{23}e^{-\beta t}\cos\omega_tt\quad x_{24}=A_{24}e^{-\beta t}\cos\omega_tt$$

Postacie drgań:

r_1

$$\left(bl^2r+kl^2\right)A_{11}=A_{21}\left(kl^2+bl^2r\right)$$

$$\frac{A_{21}}{A_{11}}=1\qquad \frac{A_{22}}{A_{12}}=1\qquad \frac{A_{23}}{A_{13}}=-1\qquad \frac{A_{24}}{A_{14}}=-1$$

r_3

$$\left(\frac{1}{3}ml^2\left(\frac{9b^2}{m^2}-i\frac{6b}{m}\sqrt{\frac{3g}{2l}+\frac{6k}{m}-\frac{9b^2}{m^2}}-\frac{3g}{2l}-\frac{6k}{m}+\frac{9b^2}{m^2}\right)+bl^2\left(-\frac{3b}{m}+i\sqrt{\frac{3g}{2l}+\frac{6k}{m}-\frac{9b^2}{m^2}}\right)+\frac{mgl}{2}+kl^2\right)A_{13}=$$
$$=\left(kl^2+bl^2\left(-\frac{3b}{m}+i\sqrt{\frac{3g}{2l}+\frac{6k}{m}-\frac{9b^2}{m^2}}\right)\right)A_{23}$$

$$\left(bl^2\left(\frac{3b}{m}-i\sqrt{\frac{3g}{2l}+\frac{6k}{m}-\frac{9b^2}{m^2}}\right)-kl^2\right)A_{13}=\left(kl^2+bl^2\left(-\frac{3b}{m}+i\sqrt{\frac{3g}{2l}+\frac{6k}{m}-\frac{9b^2}{m^2}}\right)\right)A_{23}$$
$$-\left(bl^2\left(-\frac{3b}{m}+i\sqrt{\frac{3g}{2l}+\frac{6k}{m}-\frac{9b^2}{m^2}}\right)+kl^2\right)A_{13}=\left(kl^2+bl^2\left(-\frac{3b}{m}+i\sqrt{\frac{3g}{2l}+\frac{6k}{m}-\frac{9b^2}{m^2}}\right)\right)A_{23}$$

$$x_1(t)=A_{11}\sin\omega_0 t+A_{12}\cos\omega_0 t+A_{13}e^{-\beta t}\sin\omega_t t+A_{14}e^{-\beta t}\cos\omega_t t$$
$$x_2(t)=A_{11}\sin\omega_0 t+A_{12}\cos\omega_0 t-A_{13}e^{-\beta t}\sin\omega_t t-A_{14}e^{-\beta t}\cos\omega_t t$$

$$\dot{x}_1(t)=A_{11}\omega_0\cos\omega_0 t-A_{12}\omega_0\sin\omega_0 t+A_{13}\left(\omega_t e^{-\beta t}\cos\omega_t t-\beta e^{-\beta t}\sin\omega_t t\right)+A_{14}\left(-\omega_t e^{-\beta t}\sin\omega_t t-\beta e^{-\beta t}\cos\omega_t t\right)$$
$$\dot{x}_2(t)=A_{11}\omega_0\cos\omega_0 t-A_{12}\omega_0\sin\omega_0 t-A_{13}\left(\omega_t e^{-\beta t}\cos\omega_t t-\beta e^{-\beta t}\sin\omega_t t\right)+A_{14}\left(\omega_t e^{-\beta t}\sin\omega_t t+\beta e^{-\beta t}\cos\omega_t t\right)$$

Warunki początkowe:

Przypadek 1:

$x_1(0)=0$	$A_{12}+A_{14}=0$	$A_{12}=0$	$A_{12}=0$
$x_2(0)=0$	$A_{12}-A_{14}=0$	$A_{14}=0$	$A_{14}=0$
$\dot{x}_1(0)=v_0$	$A_{11}\omega_0+A_{13}\omega_t-A_{14}\beta=v_0$	$2A_{11}\omega_0=v_0$	$A_{11}=\frac{v_0}{2\omega_0}$
$\dot{x}_2(0)=0$	$A_{11}\omega_0-A_{13}\omega_t+A_{14}\beta=0$	$2A_{13}\omega_t=v_0$	$A_{13}=\frac{v_0}{2\omega_t}$

$$\omega_0=\sqrt{\frac{3g}{2l}}\qquad \omega_t=\sqrt{\frac{3g}{2l}+\frac{6k}{m}-\frac{9b^2}{m^2}}\qquad \beta=\frac{3b}{m}$$

$$x_1(t)=\frac{v_0}{2}\left(\frac{1}{\omega_0}\sin\omega_0 t+\frac{1}{\omega_0}e^{-\beta t}\sin\omega_t t\right)$$
$$x_2(t)=\frac{v_0}{2}\left(\frac{1}{\omega_0}\sin\omega_0 t-\frac{1}{\omega_0}e^{-\beta t}\sin\omega_t t\right)$$

Przypadek 2:

$$\begin{aligned} x_1(0) &= 0 \\ x_2(0) &= 0 \\ \dot{x}_1(0) &= v_0 \\ \dot{x}_2(0) &= -v_0 \end{aligned}$$

$$\begin{aligned} A_{12} + A_{14} &= 0 \\ A_{12} - A_{14} &= 0 \\ A_{11}\omega_0 + A_{13}\omega_t - A_{14}\beta &= v_0 \\ A_{11}\omega_0 - A_{13}\omega_t + A_{14}\beta &= -v_0 \end{aligned}$$

$$\begin{aligned} A_{12} &= 0 \\ A_{14} &= 0 \\ A_{11} &= 0 \\ -A_{13}\omega_t &= -v_0 \end{aligned}$$

$$\begin{aligned} A_{12} &= 0 \\ A_{14} &= 0 \\ A_{11} &= 0 \\ A_{13} &= \frac{v_0}{\omega_t} \end{aligned}$$

$$\omega_0 = \sqrt{\frac{3g}{2l}} \qquad \omega_t = \sqrt{\frac{3g}{2l} + \frac{6k}{m} - \frac{9b^2}{m^2}} \qquad \beta = \frac{3b}{m}$$

$$x_1(t) = \frac{v_0}{\omega_t} e^{-\beta t} \sin \omega_t t$$

$$x_2(t) = \frac{v_0}{\omega_t} e^{-\beta t} \sin \omega_t t$$

Zadanie do rozwiązania!

Wyznaczyć częstości drgań własnych oraz postacie drgań belki, wspartej końcami na jednakowych układach sprężysto tłumiących o sztywności k i współczynniku tłumienia b.