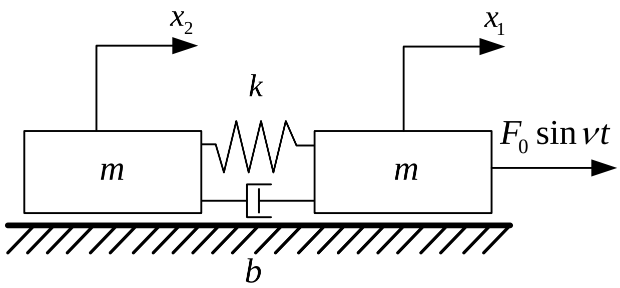
TEMAT:	**Drgania wymuszone tłumione o wielu stopniach swobody**

Przykład 1.

Sporządź charakterystyki amplitudowo-częstotliwościowe układu dwóch mas połączonych układem sprężysto-tłumiącym, zakładając że mają równe masy m.



$$E = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2$$

$$V = \frac{1}{2} k \left(x_1 - x_2\right)^2$$

$$D = \frac{1}{2} b \left(\dot{x}_1 - \dot{x}_2\right)^2$$

$$m_1 \ddot{x}_1 + b\left(\dot{x}_1 - \dot{x}_2\right) + k\left(x_1 - x_2\right) = F_0 \sin \nu t$$

$$m_2 \ddot{x}_2 - b\left(\dot{x}_1 - \dot{x}_2\right) - k\left(x_1 - x_2\right) = 0$$

$$\ddot{x}_1 + 2\beta\left(\dot{x}_1 - \dot{x}_2\right) + \omega_0^2\left(x_1 - x_2\right) = f_0 \sin \nu t$$

$$\ddot{x}_2 - 2\beta\left(\dot{x}_1 - \dot{x}_2\right) - \omega_0^2\left(x_1 - x_2\right) = 0$$

gdzie: $\beta = \frac{b}{2m} \quad \omega_0 = \sqrt{\frac{k}{m}} \quad f = \frac{F_0}{m}$

$$x_1 = A_1 e^{i\nu t} \qquad \dot{x}_1 = iA_1 \nu e^{i\nu t} \qquad \ddot{x}_1 = -A_1 \nu^2 e^{i\nu t}$$

$$x_2 = A_2 e^{i\nu t} \qquad \dot{x}_2 = iA_2 \nu e^{i\nu t} \qquad \ddot{x}_2 = -A_2 \nu^2 e^{i\nu t}$$

gdzie: $e^{-i\omega t} = \cos\left(\omega t\right) - i \sin\left(\omega t\right)$

$$A_1\left(-\nu^2 + i2\beta\nu + \omega_0^2\right) - A_2\left(i2\beta\nu + \omega_0^2\right) = f_0$$

$$-A_1\left(i2\beta\nu + \omega_0^2\right) + A_2\left(-\nu^2 + i2\beta\nu + \omega_0^2\right) = 0$$

$$\begin{bmatrix} \left(-\nu^2 + i2\beta\nu + \omega_0^2\right) & -\left(i2\beta\nu + \omega_0^2\right) \\ -\left(i2\beta\nu + \omega_0^2\right) & \left(-\nu^2 + i2\beta\nu + \omega_0^2\right) \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \begin{bmatrix} f_0 \\ 0 \end{bmatrix}$$

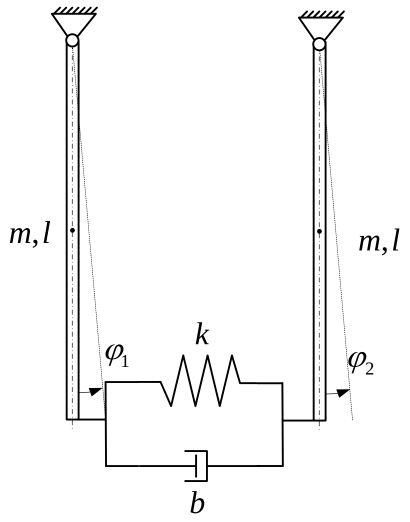
$$W = \begin{vmatrix} \left(-\nu^2 + i2\beta\nu + \omega_0^2\right) & -\left(i2\beta\nu + \omega_0^2\right) \\ -\left(i2\beta\nu + \omega_0^2\right) & \left(-\nu^2 + i2\beta\nu + \omega_0^2\right) \end{vmatrix} = r^2\left(r^2 + 4\beta r + 2\omega_0^2\right) = -\nu^2\left(-\nu^2 + i4\beta\nu + 2\omega_0^2\right)$$

$$W_1 = \begin{vmatrix} f_0 & -\left(i2\beta\nu + \omega_0^2\right) \\ 0 & \left(-\nu^2 + i2\beta\nu + \omega_0^2\right) \end{vmatrix} = f_0\left(-\nu^2 + i2\beta\nu + \omega_0^2\right)$$

$$W_2 = \begin{vmatrix} \left(-\nu^2 + i2\beta\nu + \omega_0^2\right) & f_0 \\ -\left(i2\beta\nu + \omega_0^2\right) & 0 \end{vmatrix} = f_0\left(i2\beta\nu + \omega_0^2\right)$$

$$\left|A_1\right| = \left|\frac{W_1}{W}\right| = \frac{f_0}{\nu^2} \frac{\sqrt{\left(\omega_0^2 - \nu^2\right)^2 + 4\beta^2\nu^2}}{\sqrt{\left(2\omega_0^2 - \nu^2\right)^2 + 16\beta^2\nu^2}}$$

$$\left|A_2\right| = \left|\frac{W_2}{W}\right| = \frac{\left|f_0\omega_0^2 + f_0 2\beta\nu i\right|}{\left|-\nu^2\left(-\nu^2 + \omega_0^2\right) - i2\beta\nu^3\right|} = \frac{f_0}{\nu^2} \frac{\sqrt{\omega_0^4 + 4\beta^2\nu^2}}{\sqrt{\left(\omega_0^2 - \nu^2\right)^2 + 4\beta^2\nu^2}}$$



$$E=\frac{1}{2}\frac{1}{3}ml^2\left(\dot{\varphi}_1^2+\dot{\varphi}_2^2\right)$$

$$V=-mg\frac{l}{2}\left(\cos\varphi_1+\cos\varphi_2\right)+\frac{1}{2}kl^2\left(\varphi_1-\varphi_2\right)^2$$

$$D=\frac{1}{2}bl^2\left(\dot{\varphi}_1-\dot{\varphi}_2\right)^2$$

$$\frac{1}{3}ml^2\ddot{\varphi}_1+\frac{mgl}{2}\varphi_1+bl^2\left(\dot{\varphi}_1-\dot{\varphi}_2\right)+kl^2\left(\varphi_1-\varphi_2\right)=M_0\sin\nu t$$

$$\frac{1}{3}ml^2\ddot{\varphi}_2+\frac{mgl}{2}\varphi_2-bl^2\left(\dot{\varphi}_1-\dot{\varphi}_2\right)-kl^2\left(\varphi_1-\varphi_2\right)=0$$

$$\ddot{\varphi}_1+\frac{3g}{2l}\varphi_1+\beta\left(\dot{\varphi}_1-\dot{\varphi}_2\right)+\frac{3k}{m}\left(\varphi_1-\varphi_2\right)=m_0e^{i\nu t}$$

$$\ddot{\varphi}_2+\frac{3g}{2l}\varphi_2-\beta\left(\dot{\varphi}_1-\dot{\varphi}_2\right)-\frac{3k}{m}\left(\varphi_1-\varphi_2\right)=0$$

gdzie: $m_0=\frac{3M_0}{ml^2}$, $\beta=\frac{3b}{m}$

$$\varphi_1=A_1e^{i\nu t}\qquad \dot{\varphi}_1=iA_1\nu e^{i\nu t}\qquad \ddot{\varphi}_1=-A_1\nu^2e^{i\nu t}$$

$$\varphi_2=A_2e^{i\nu t}\qquad \dot{\varphi}_2=iA_2\nu e^{i\nu t}\qquad \ddot{\varphi}_2=-A_2\nu^2e^{i\nu t}$$

$$\boldsymbol{M}\ddot{\boldsymbol{\varphi}}+\boldsymbol{B}\dot{\boldsymbol{\varphi}}+\boldsymbol{K}\boldsymbol{\varphi}=0$$

$\omega_1=\sqrt{\frac{3g}{2l}}$	$\omega_2=\sqrt{\frac{3g}{2l}+\frac{6k}{m}}$

$$\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}\ddot{\varphi}_1\\\ddot{\varphi}_2\end{bmatrix}+\begin{bmatrix}\beta&-\beta\\-\beta&\beta\end{bmatrix}\begin{bmatrix}\dot{\varphi}_1\\\dot{\varphi}_2\end{bmatrix}+\begin{bmatrix}\frac{3g}{2l}+\frac{3k}{m}&-\frac{3k}{m}\\-\frac{3k}{m}&\frac{3g}{2l}+\frac{3k}{m}\end{bmatrix}\begin{bmatrix}\varphi_1\\\varphi_2\end{bmatrix}=\begin{bmatrix}m_0e^{i\nu t}\\0\end{bmatrix}$$

$$\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}\ddot{\varphi}_1\\\ddot{\varphi}_2\end{bmatrix}+\begin{bmatrix}\beta&-\beta\\-\beta&\beta\end{bmatrix}\begin{bmatrix}\dot{\varphi}_1\\\dot{\varphi}_2\end{bmatrix}+\begin{bmatrix}a+b&-b\\-b&a+b\end{bmatrix}\begin{bmatrix}\varphi_1\\\varphi_2\end{bmatrix}=\begin{bmatrix}m_0e^{i\nu t}\\0\end{bmatrix}$$

$$a=\frac{3g}{2l},b=\frac{3k}{m}$$

$$-\nu^2\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}A_1\\A_2\end{bmatrix}+i\nu\begin{bmatrix}\beta&-\beta\\-\beta&\beta\end{bmatrix}\begin{bmatrix}A_1\\A_2\end{bmatrix}+\begin{bmatrix}a+b&-b\\-b&a+b\end{bmatrix}\begin{bmatrix}A_1\\A_2\end{bmatrix}=\begin{bmatrix}m_0\\0\end{bmatrix}$$

$$\begin{bmatrix}-\nu^2+a+b+i\beta\nu&-\left(b+i\beta\nu\right)\\-\left(b+i\beta\nu\right)&-\nu^2+a+b+i\beta\nu\end{bmatrix}\begin{bmatrix}A_1\\A_2\end{bmatrix}=\begin{bmatrix}m_0\\0\end{bmatrix}$$

$$W = \left(-\nu^2 + a + b + i\beta\nu\right)^2 - \left(b + i\beta\nu\right)^2 = \left(-\nu^2 + a + b\right)^2 + 2\left(-\nu^2 + a + b\right)i\beta\nu - \beta^2\nu^2 - b^2 - i2\beta b\nu + \beta^2\nu^2$$

$$W = \left(-\nu^2 + a + b\right)^2 + 2\left(-\nu^2 + a\right)i\beta\nu - b^2$$

$$W = -\nu^4 + a^2 + b^2 - 2\nu^2 a - 2\nu^2 b + 2ab - b^2 + i2\beta\nu\left(-\nu^2 + a\right)$$

$$W = \left(-\nu^2 + a\right)^2 - 2\nu^2 b + 2ab + i2\beta\nu\left(-\nu^2 + a\right)$$

$$W_1 = m_0\left(-\nu^2 + a + b + i\beta\nu\right)$$

$$W_2 = m_0\left(b + i\beta\nu\right)$$

$$A_1 = \frac{m_0\left(-\nu^2 + a + b + i\beta\nu\right)}{\left(-\nu^2 + a\right)^2 - 2\nu^2 b + 2ab + i2\beta\nu\left(-\nu^2 + a\right)}$$

$$\left|A_1\right| = m_0 \frac{\sqrt{\left(-\nu^2 + a + b\right)^2 + \beta^2\nu^2}}{\sqrt{\left(\left(-\nu^2 + a\right)^2 - 2\nu^2 b + 2ab\right)^2 + 4\beta^2\nu^2\left(-\nu^2 + a\right)^2}}$$

$$\left|A_2\right| = m_0 \frac{\sqrt{\left(-\nu^2 + a + b\right)^2 + \beta^2\nu^2}}{\sqrt{\left(\left(-\nu^2 + a\right)^2 - 2\nu^2 b + 2ab\right)^2 + 4\beta^2\nu^2\left(-\nu^2 + a\right)^2}}$$