Optimal Sensor Placement for Identification of Electrical Networks using Noisy Measurement Data

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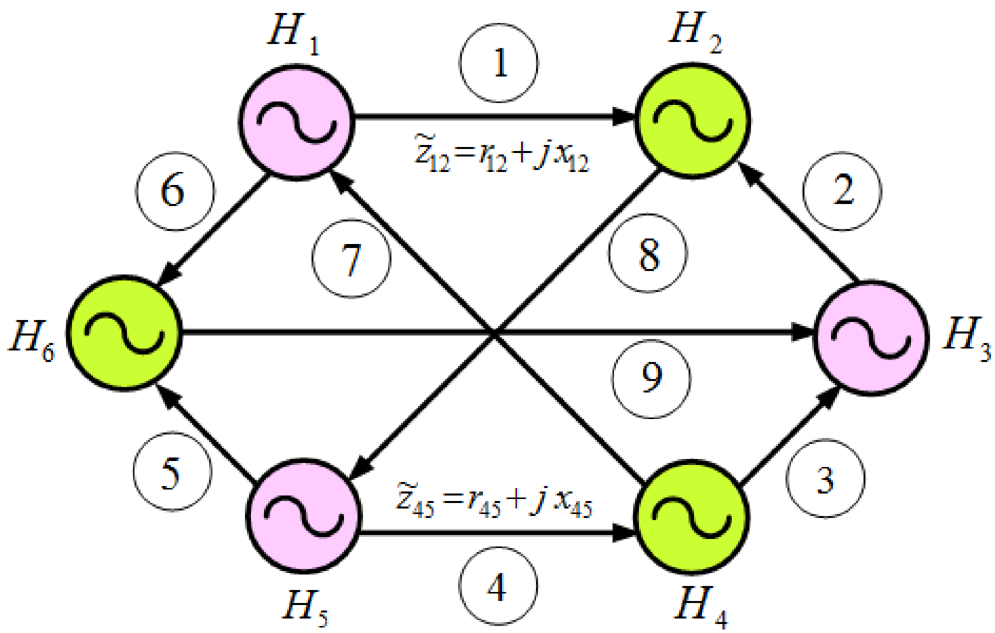
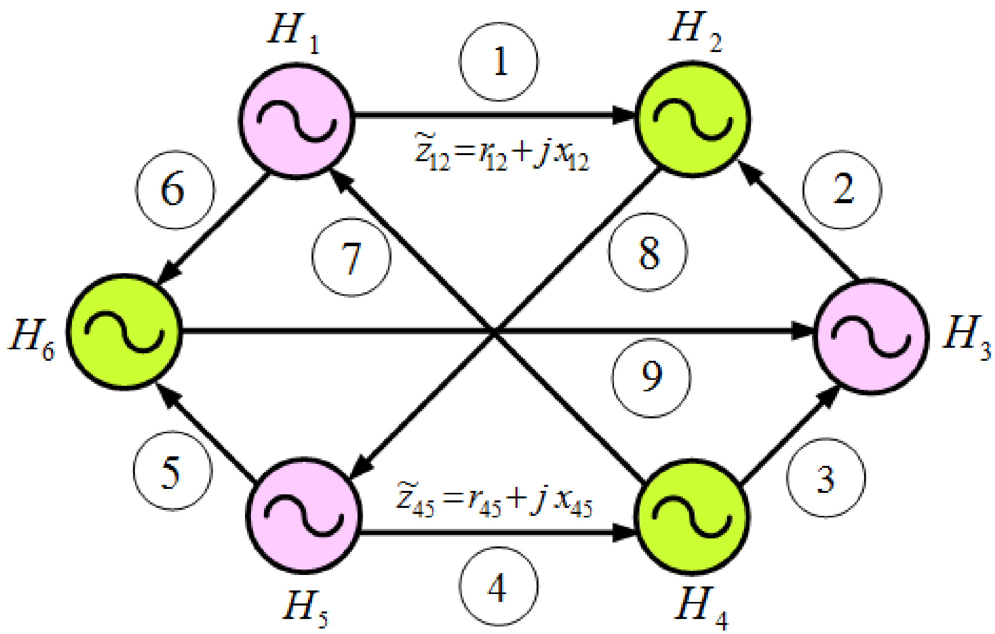
Network Model Identification

Assuming that voltage, phase and frequency measurements from the terminal buses of every tie-line of the power network are available (directly or by observability):

How to choose the best set of (virtual) measurement locations on the tie-lines to generate accurate second-order models, especially when the PMU data are noisy and unreliable



Network Model Identification



$$p_{ik} = r_{ik}^{\ 2} + x_{ik}^{\ 2}, \quad \alpha_{ik} = \tan^{-1}(x_{ik} / r_{ik})$$

$$\dot{\delta}_i = \omega_i$$

$$2H_i \dot{\omega}_i = P_{mi} - \sum_{k \in N_i} \left(\frac{E_i^{\ 2} r_{ik} - E_i E_k p_{ik} \cos(\delta_{ik} + \alpha_{ik})}{p_{ik}^{\ 2}} \right)$$

2nd order oscillators connected over a planar network graph

Nodes represent electrical generators, Edges denote the intertie lines

Edges have impedances with a real resistive part and an imaginary reactive part

Dynamics of each generator are driven by power balance with its neighbors following Newton's law

Assume direct connection between the oscillators implying that the graph is a connected differential network (and not a differential–algebraic network)

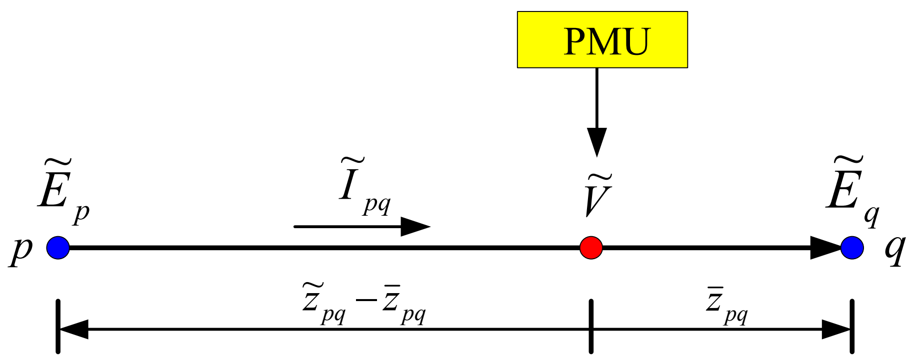
Identification of Network Model Parameters

$$\boxed{\begin{aligned} &\dot{\delta}_i = \omega_i \\ &2H_i\dot{\omega}_i = P_{mi} - \sum_{k\in N_i}\left(\frac{E_i^{\,2} r_{ik} - E_i E_k p_{ik}\cos(\delta_{ik} + \alpha_{ik})}{p_{ik}^{\,2}}\right) \end{aligned}}$$

$$\delta_{ik} = \delta_i - \delta_k, \quad p_{ik} = r_{ik}^{\,2} + x_{ik}^{\,2}, \quad \alpha_{ik} = \tan^{-1}(x_{ik}/r_{ik})$$

Problem:

How to estimate the parameters: $E_p \quad E_q \quad H_p \quad H_q \quad r_{pq} \quad x_{pq}$



$$E_p = \mathrm{Re}(\tilde{V} - \bar{z}_{pq}\tilde{I})$$

$$E_q = \mathrm{Re}(\tilde{V} + (\tilde{z}_{pq} - \bar{z}_{pq})\tilde{I})$$

Estimate: $H_p \quad H_q \quad r_{pq} \quad x_{pq}$

Identification of Network Model Parameters

Small-signal Model

$$\begin{bmatrix} \Delta\dot{\delta} \\ \Delta\dot{\omega} \end{bmatrix} = \begin{bmatrix} 0 & I \\ \mathcal{L} & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \end{bmatrix} + \begin{bmatrix} 0 \\ \mathcal{E} \end{bmatrix} u$$

Laplacian Structure

$$\mathcal{L}_{pp} = -\sum_{q \in N_p}^{n} \frac{E_p E_q}{2H_p z_{pq}} \sin(\delta_{pq0} + \alpha_{pq0})$$

$$\mathcal{L}_{pq} = \frac{E_p E_q}{2H_p z_{pq}} \sin(\delta_{pq0} + \alpha_{pq0}), \quad q \in N_p$$

$$\mathcal{L}_{pq} = 0, \quad \text{otherwise}$$

- Straightforward in the deterministic case
- Standard Least-squares methods (ARMA) can be applied
- Linearize the swing model, apply an impulse input, observe the output, estimate parameters from i/o identification

Identification of Network Model Parameters

Small-signal Model

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- Straightforward in the deterministic case
- Standard Least-squares methods (ARMA) can be applied
- Linearize the swing model, apply an impulse input, observe the output, estimate parameters from i/o identification

- However, if PMU measurements are noisy - parameter estimates are probabilistic
- Estimation must be posed in terms of estimation error bounds – Cramer Rao bounds

Statistical Preliminaries

- Given a data sequence *y(t)*, we consider the estimator *g(y)* to estimate the parameter vector θ

- Let the Fisher Information Matrix (FIM) for the θ be $J(\theta)$

- Estimation error covariance:

$$C_g = E[(g(y)-\theta)(g(y)-\theta)^T]$$

- Cramer Rao bound: $C_g \le J^{-1}(\theta)$

- For LTI systems (McWhorter & Scharf):

$$y = x(\theta) + n \qquad n : N[0,R]$$

$$J(\theta) = \left(\frac{\partial x}{\partial \theta}\right) R^{-1} \left(\frac{\partial x}{\partial \theta}\right)^T$$

$$\Rightarrow$$

$$\theta = col(a,b)$$

$$J(a,b) = \frac{1}{\sigma^2}\begin{bmatrix} HH^T & HK^T \\ KH^T & KK^T \end{bmatrix}$$

where:

$$R = \sigma^2 I$$

$$H = \frac{\partial x}{\partial a} \quad K = \frac{\partial x}{\partial b}$$

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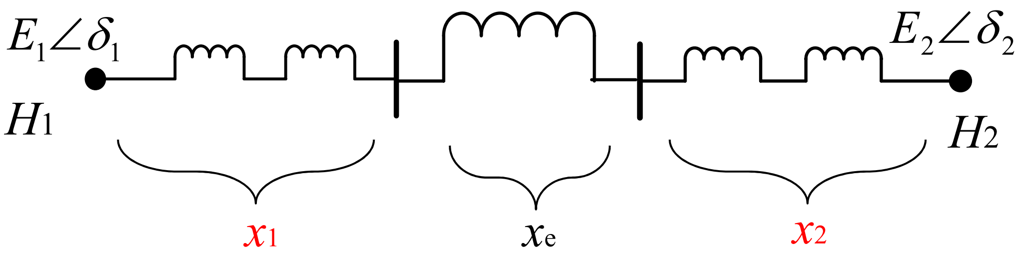
➡ For our model the FIM *J(a, b)* is a function of the spatial location of the PMU

➡ There exists an `optimal location' that generates the tightest CRB

➡ If there is no internal output feedback on any point of the edge from the terminal buses, then computation is decentralized (CDC 2010)

- for example, if there is a SVC for voltage regulation then a centralized algorithm is needed

Two-node System

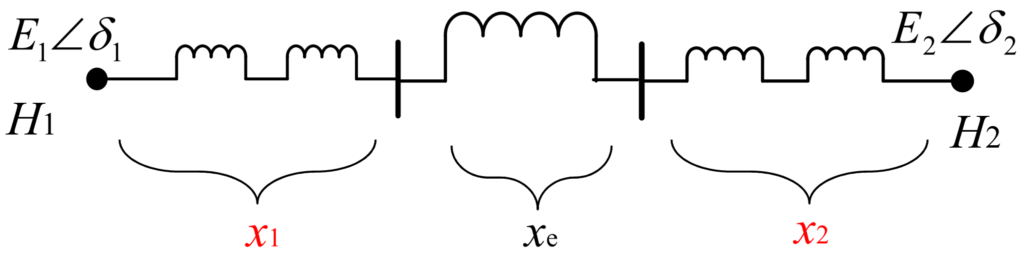


$$\dot{\delta} = \omega$$

$$2\frac{H_1H_2}{H_1+H_2}\dot{\omega} = \frac{H_2P_{m1} - H_1P_{m2}}{H_1+H_2} - \frac{E_1E_2}{(x_1+x_e+x_2)}\sin\delta$$

- Linearize: $\dot{m} = \begin{bmatrix} 0 & 1 \\ \frac{-E_1E_2}{2H\bar{x}}\cos(\delta_o) & 0 \end{bmatrix} m + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \qquad m = [\Delta\delta, \Delta\omega], \qquad 0 < \delta_0 < 90^o$

Two-node System



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- Assume output as voltage magnitude V measured at distance x from Gen 2

$$a = x / \bar{x}$$
$$\bar{x} = x_1 + x_e + x_2$$

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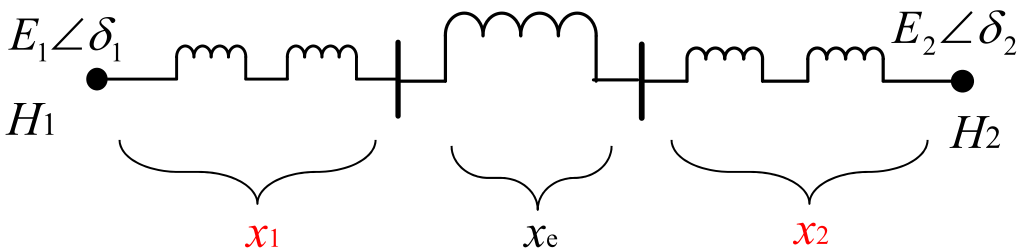

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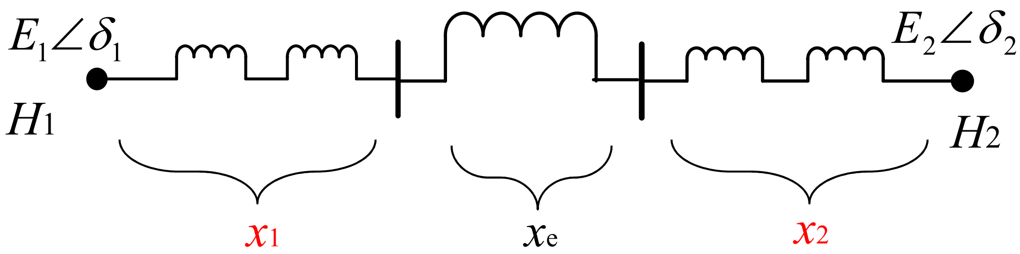
$$a = x / \bar{x}$$

$$\bar{x} = x_1 + x_e + x_2$$

- From Ohm's law: $V = \sqrt{E_2{}^2(1-a)^2 + E_1{}^2 a^2 + 2E_1E_2 a(1-a)\cos(\delta)}$

- Output matrix: $C = \begin{bmatrix} \frac{\partial V}{\partial \delta} & \frac{\partial V}{\partial \omega} \end{bmatrix}\Big|_{\delta_0, 0} = \begin{bmatrix} \frac{-a(1-a)E_1E_2\sin(\delta_0)}{V_0(a)} & 0 \end{bmatrix}$ ⇒ Function of a

Two-node System



Discrete-time Transfer Function

$$G(z) = \psi(a)\frac{K(z+1)}{z^2 - 2\cos(\gamma T)z + 1},$$

$$\psi(a) = a(1-a)/V_0(a)$$

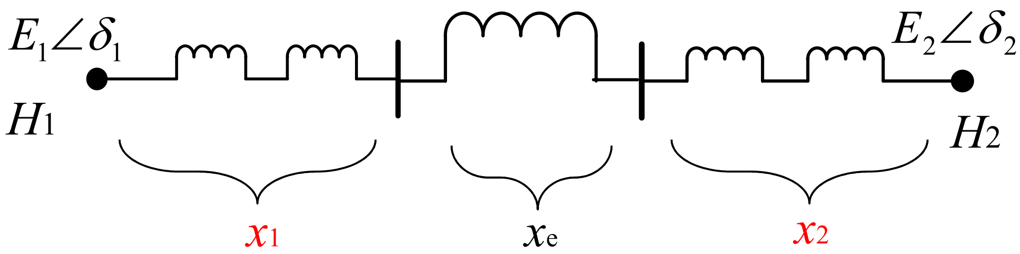
$\Rightarrow$

Voltage Magnitude as Output

$$V(k,a) = \psi(a)\xi(\underbrace{k, x_1, x_2, H_1, H_2}_{\textit{unknown parameters}}) \qquad k = 1, 2, 3, \ldots$$

$\alpha = \{x_1, x_2\}$ $\quad$ $\beta = \{H_1, H_2\}$

Two-node System



Discrete-time Transfer Function

$$G(z)=\psi(a)\frac{K(z+1)}{z^2-2\cos(\gamma T)z+1},$$

$$\psi(a)=a(1-a)/V_0(a)$$

$\Rightarrow$

Voltage Magnitude as Output

$$V(k,a)=\psi(a)\underbrace{\xi(k,x_1,x_2,H_1,H_2)}_{unknown\ parameters} \qquad k=1,\ 2,\ 3,\ldots.$$

$\alpha=\{x_1,x_2\} \qquad \beta=\{H_1,H_2\}$

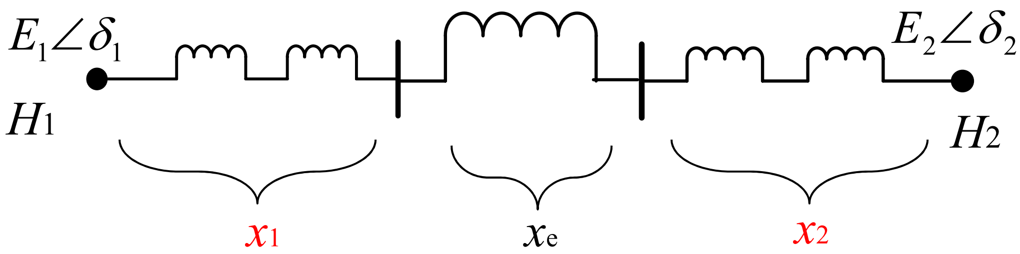
Construction of FIM

- Stack:

$$Y(a,k)=col[V(1),V(2),\ldots,V(k)] \qquad \alpha=\{x_1,x_2\} \qquad \beta=\{H_1,H_2\}$$

$$\boxed{H(a)=\frac{\partial Y}{\partial \alpha} \qquad K(a)=\frac{\partial Y}{\partial \beta}} \Rightarrow J(a,\alpha,\beta)=\frac{1}{\sigma^2}\begin{bmatrix} HH^T & HK^T \\ KH^T & KK^T \end{bmatrix} \Rightarrow \text{Function of } a$$

Two-node System



- Recall: Minimizing the estimation error bound = min J^{-1}
- Equivalently, for any given set of measurements, maximize det(J)
- Main Result: Find a such that the determinant of $J(a)$ is maximised

$$J(a,\alpha,\beta) = \begin{vmatrix} HH^T & HK^T \\ KH^T & KK^T \end{vmatrix}$$

$$a \in [a_1, a_2]$$

$$a_1 = \frac{x_2}{x_1 + x_e + x_2} \quad a_2 = \frac{x_2 + x_e}{x_1 + x_e + x_2}$$

$$\max \psi^2 \det(\psi_x{}^2 L_1 + \psi_x \psi L_2 + \psi^2 L_3)$$
$$st.x \in [x_2, x_e + x_2]$$

PMU's can only be placed on the transmission line, not inside the areas

Estimation Algorithm

Algorithm 1 Calculate $\mathcal{A}^*$ for model (4)-(5)

Choose $\mathcal{A}(i) \in [0,1], \; i = 1,2,\ldots,m$
$f = 1, \; 0 < \epsilon^* \ll 1$
if $f = 1$ **then**
  Measure: $\tilde{y}_j(a_j, k), \; k = 1,2,\ldots,l, \; j = 1,2,\ldots,m$
  Solve:

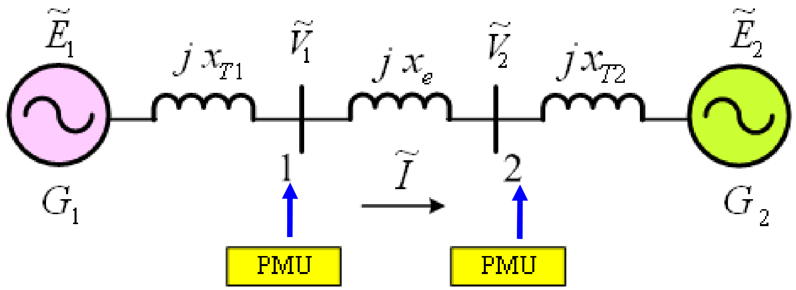
$$\hat{\Theta}_{ML} = \max_{\Theta} \sum_{j=1}^{m} \sum_{k=1}^{l} \left\{ \tilde{y}_j(\mathcal{A}(j), k) * y_j(\mathcal{A}(j), k, \Theta) \right\}$$

$$\mathcal{A}^* = \max_{\mathcal{A}} \det(J_(\hat{\Theta}_{ML}, \mathcal{A})), \quad \mathcal{A}(i) \in [0,\, 1]$$

  Set $\epsilon = ||\mathcal{A} - \mathcal{A}^*||_2$
  Set $\mathcal{A} = \mathcal{A}^*$
  if $\epsilon \leq \epsilon^*$ **then**
    $f = 0$
  else
    $f = 1$
  end if
end if

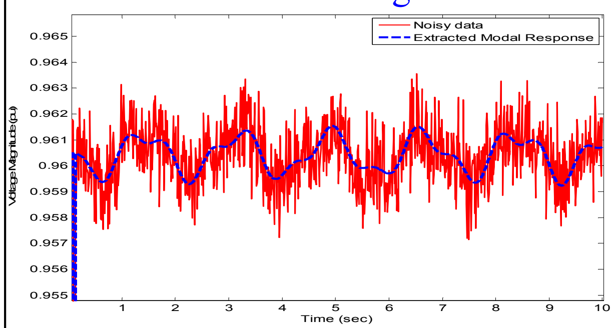
⇨ Local convexity in *a*
⇨
⇨ Global convexity needs uniformity in all parameters in ***S***

Simulations

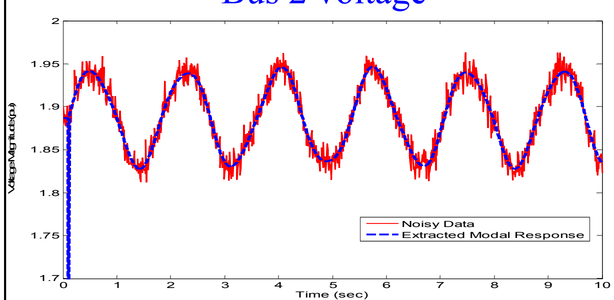


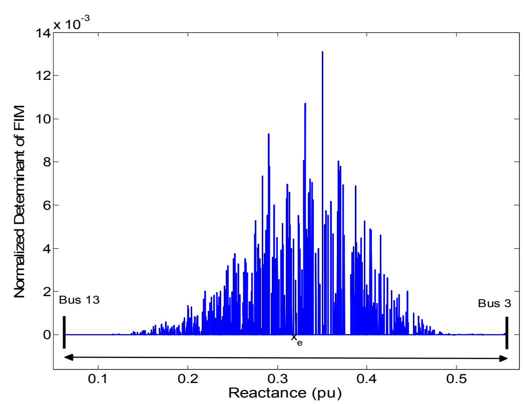
Parameters	Actual Values	Iteration 1	Iteration 1	Iteration 1	Iteration 1	Iteration 1
r	0.1	0.05	0.06	0.08	0.08	0.093
x	1	0.58	0.64	0.78	0.83	0.981
H_1	19	15.65	17.91	17.65	18.55	18.98
H_2	13	10.86	10.91	11.68	12.35	12.75
a	-	0.428	0.412	0.409	0.408	0.408

Bus 1 voltage



Bus 2 voltage

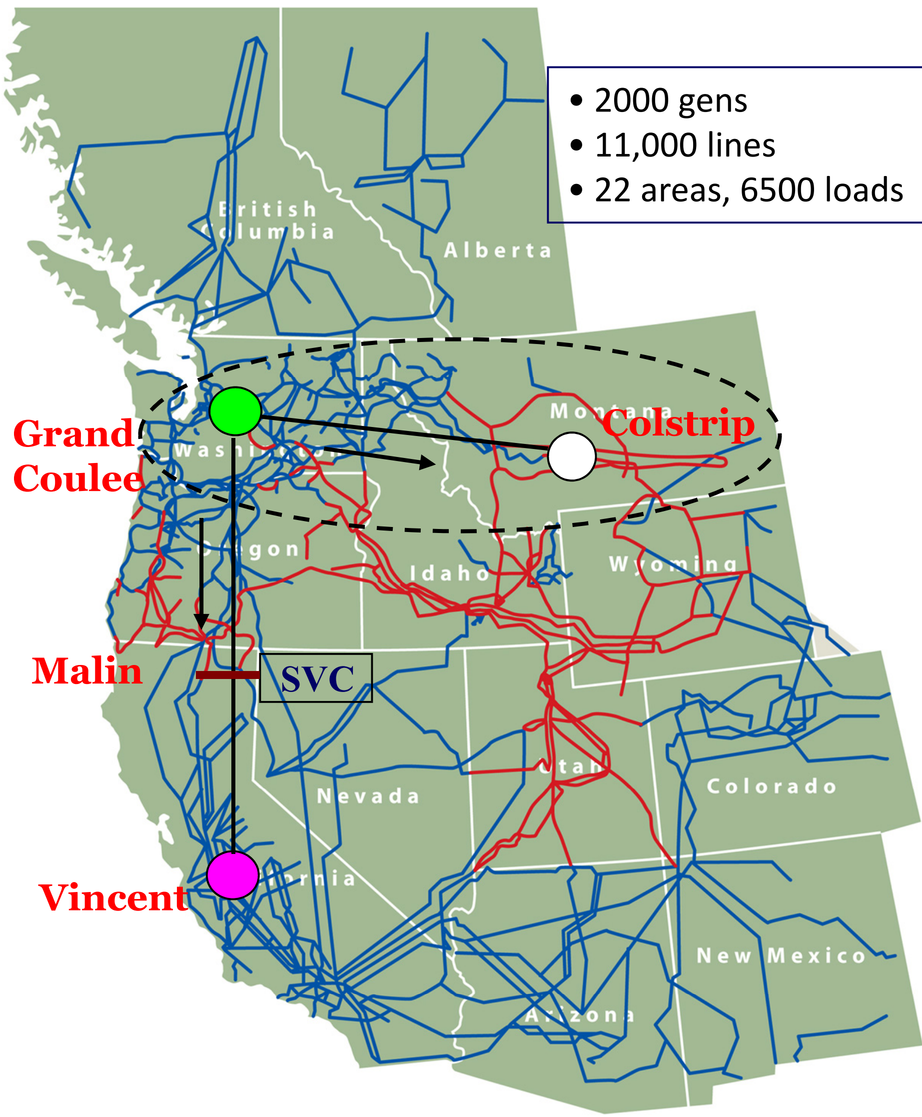


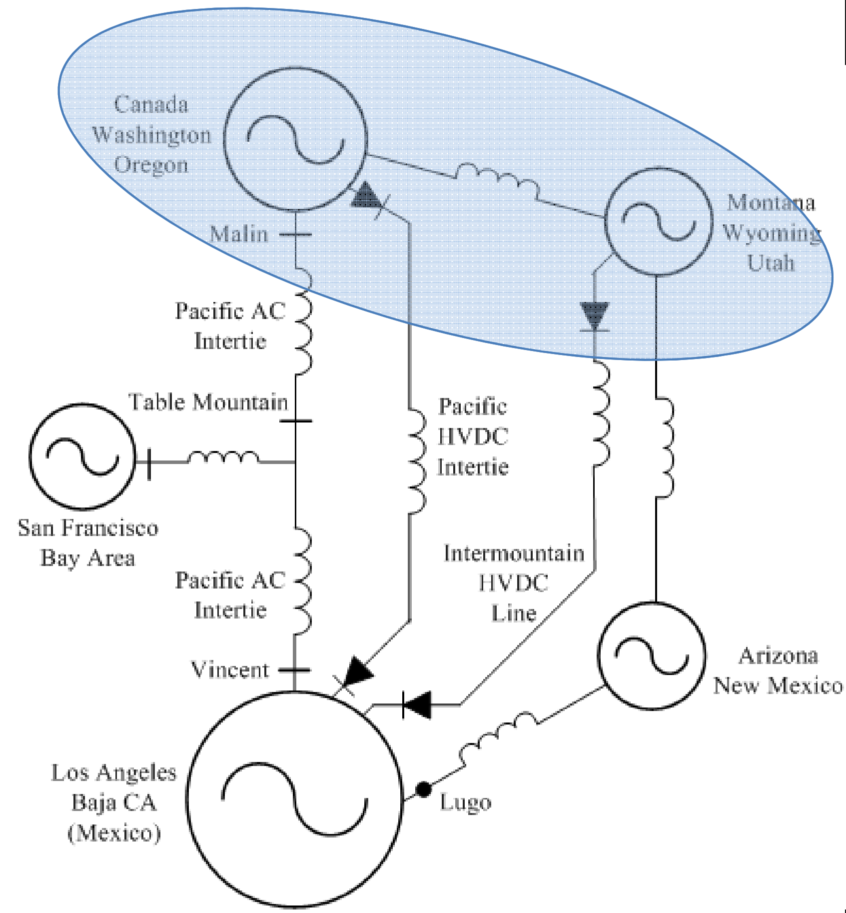


Spatial variation of the determinant of the FIM

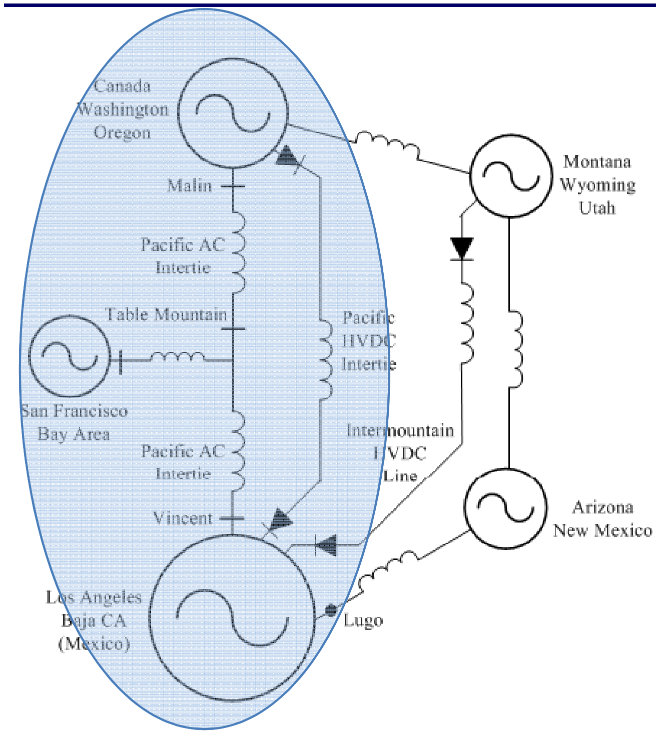
Application to WECC Data

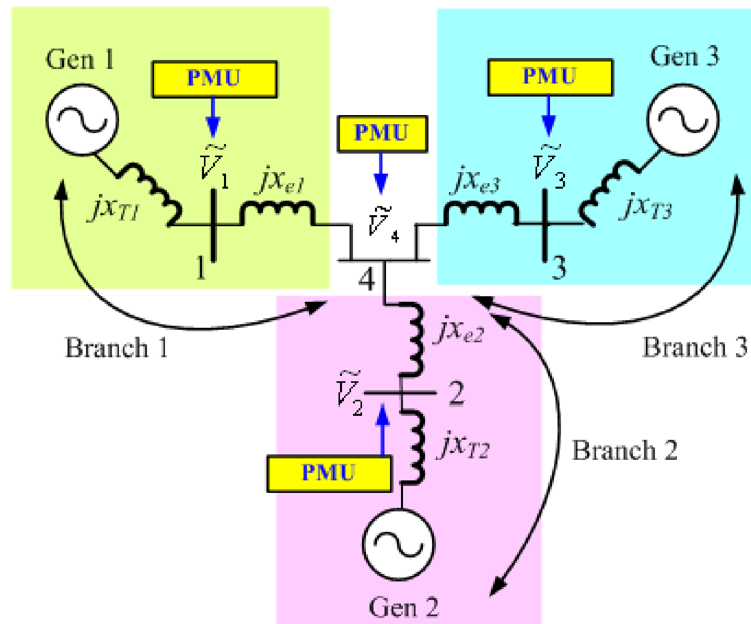
- 2000 gens
- 11,000 lines
- 22 areas, 6500 loads





More than Two Areas: Pacific AC Intertie





Salient Points

- Current in each branch is different
- No *single* spatial variable *a*
- Derivations need to be done *piecewise* (each edge of the star)
- Two interarea modes/ relative states – δ_1 & δ_2

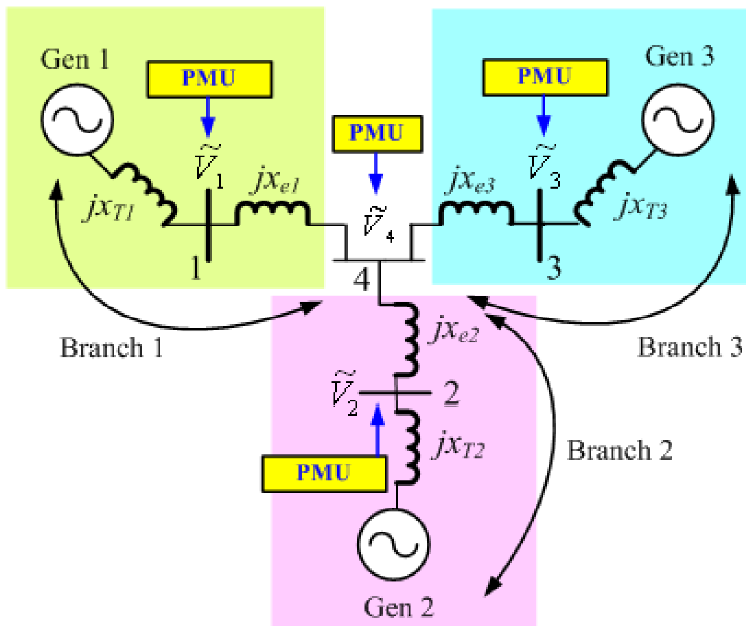
$$V_n = J_1(x)\Delta\delta_1(t) + J_2(x)\Delta\delta_2(t)$$

$$\frac{V_n^{\,3}}{V_n^{\,4}} = \frac{J_1^{\,3}(x)\Delta\delta_1(t^*) + J_2^{\,3}(x)\Delta\delta_2(t^*)}{J_1^{\,4}(x)\Delta\delta_1(t^*) + J_2^{\,4}(x)\Delta\delta_2(t^*)}$$

Time-space separation property lost!

- *Chakrabortty & Salazar* (2009, 2010)

Pacific AC Intertie



Solution – Use phase angle as a 2nd degree of freedom

$$\theta = \tan^{-1}\left(\frac{f_1(x)\sin(\delta_1) + f_2(x)\sin(\delta_2)}{f_1(x)\cos(\delta_1) + f_2(x)\cos(\delta_1) + f_3(x)}\right)$$

$$\Delta\theta(t) = S_1(x)\Delta\delta_1(t) + S_2(x)\Delta\delta_2(t)$$

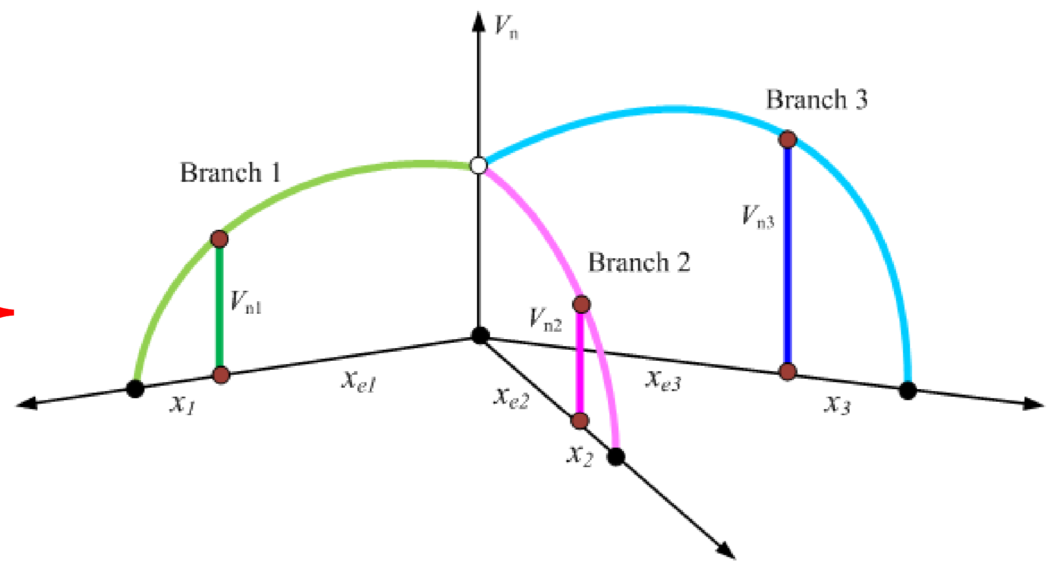
Measurable if a PMU
is installed at that point

Voltage equation

$$\frac{V_n^{\ 3}}{V_n^{\ 4}} = \frac{J_1^{\ 3}(x)\Delta\delta_1(t^*) + J_2^{\ 3}(x)\Delta\delta_2(t^*)}{J_1^{\ 4}(x)\Delta\delta_1(t^*) + J_2^{\ 4}(x)\Delta\delta_2(t^*)}$$

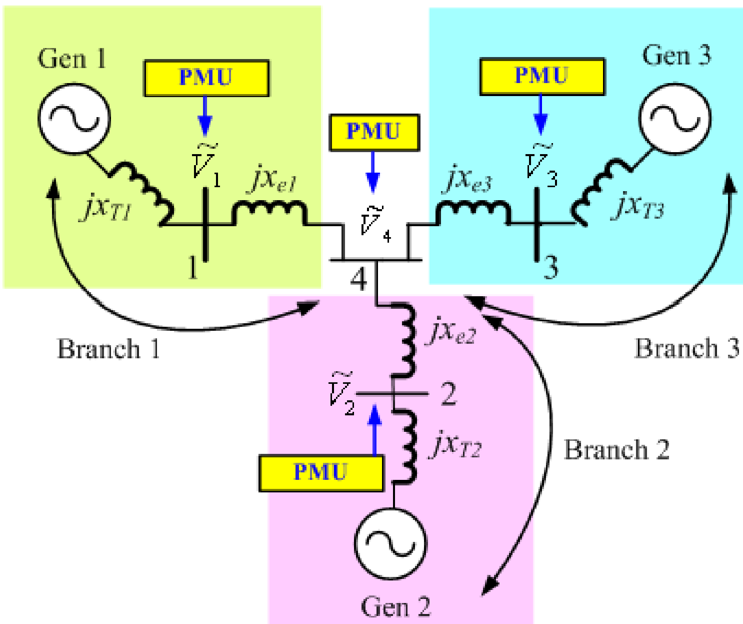
Phase equation

$$\frac{\Delta\theta^3}{\Delta\theta^4} = \frac{S_1^{\ 3}(x)\Delta\delta_1(t^*) + S_2^{\ 3}(x)\Delta\delta_2(t^*)}{S_1^{\ 4}(x)\Delta\delta_1(t^*) + S_2^{\ 4}(x)\Delta\delta_2(t^*)}$$



Adding Phase & Frequency Information in the Output

- For any branch e_{pq}



Phase

$$\theta_{pq} = \tan^{-1}\frac{a_{pq}E_p\sin(\delta_p) + (1-a_{pq})E_q\sin(\delta_q)}{a_{pq}E_p\cos(\delta_p) + (1-a_{pq})E_q\cos(\delta_q)}$$

$$\Delta\theta_{pq}(k) = \underbrace{Z^{-1}\left(C^{\theta}_{pq}(zI - A_d)^{-1}B_d\right)}_{\rho(a_{pq},k)}$$

Frequency

$$f_{pq} = \dot{\theta}_{pq} = \frac{\partial\theta_{pq}}{\partial\delta_p}\omega_p + \frac{\partial\theta_{pq}}{\partial\delta_q}\omega_q = C_1\omega_p + C_2\omega_q$$

$$\Delta f_{pq}(k) = \underbrace{Z^{-1}\left(C^{f}_{pq}(zI - A_d)^{-1}B_d\right)}_{\vartheta(a_{pq},k)}$$

Use a linear combination of outputs

$$y(k) = \mu_1\left(\psi(a_{pq})\xi(k)\right) + \mu_2\rho(a_{pq},k) + \mu_2\vartheta(a_{pq},k)$$

$$\mu_1 + \mu_2 + \mu_3 = 1$$

Adding Phase & Frequency

For any edge e_{pq} :

1. Voltage Magnitude (volts)

$$V(k,a) = \psi_1(a)\xi_1(k,r,x,H_1,H_2)$$

2. Voltage Phase Angle (rad)

$$\theta(k,a) = \psi_2(a)\xi_2(k,x,r,H_1,H_2)$$

3. Bus Frequency (rad/sec)

$$f(k,a) = \psi_3(a)\xi_3(k,R,x,H_1,H_2)$$

$k = 1, 2, 3, \ldots.$

Combined output:

$$y(k,a) = \mu_1 V(k,a) + \mu_2\theta(k,a) + \mu_3 f(k,a)$$

$$\mathbf{H}_{pq} = \frac{\partial y_{pq}}{\partial \mathbf{s}_1} = \mu_1\left(\frac{\partial \psi(a_{pq})}{\partial \mathbf{s}_1}\xi(k) + \psi(a_{pq})\frac{\partial \xi(k)}{\partial \mathbf{s}_1}\right) + \mu_2\frac{\partial \rho(a_{pq},k)}{\partial \mathbf{s}_1} + \mu_3\frac{\partial \vartheta(a_{pq},k)}{\partial \mathbf{s}_1}$$

$$\mathbf{K}_{pq} = \frac{\partial y_{pq}}{\partial \mathbf{s}_2} = \mu_1\psi(a_{pq})\frac{\partial \xi(k)}{\partial \mathbf{s}_2} + \mu_2\frac{\partial \rho(a_{pq},k)}{\partial \mathbf{s}_2} + \mu_3\frac{\partial \vartheta(a_{pq},k)}{\partial \mathbf{s}_2}$$

For exact expressions of the spatial functions please see

Chakrabortty & Martin, CDC 2010, Atlanta

$$\max \quad \det(J(a,\mu,\alpha,\beta)) = \det\begin{bmatrix} HH^T & HK^T \\ KH^T & KK^T \end{bmatrix}$$

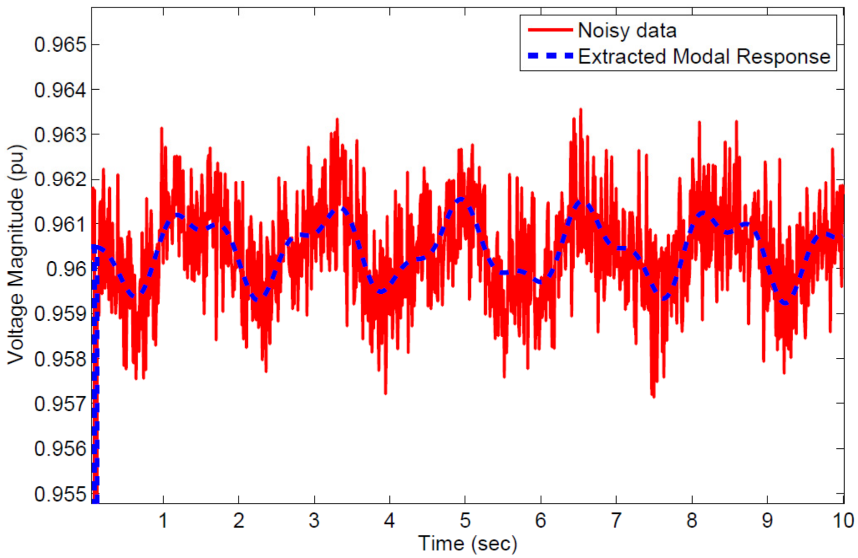
$$a \in [a_1, a_2] \quad \mu_1 > 0, \mu_2 > 0, \mu_3 > 0$$

$$a_1 = \frac{x_2}{x_1 + x_e + x_2} \qquad a_2 = \frac{x_2 + x_e}{x_1 + x_e + x_2}$$

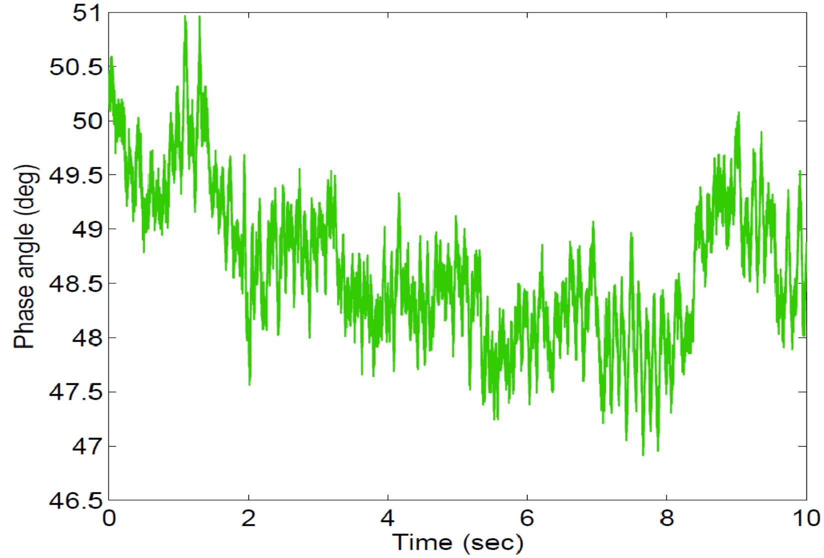
Find optimal location of sensor, and also optimal contribution of measurement variables

Simulations

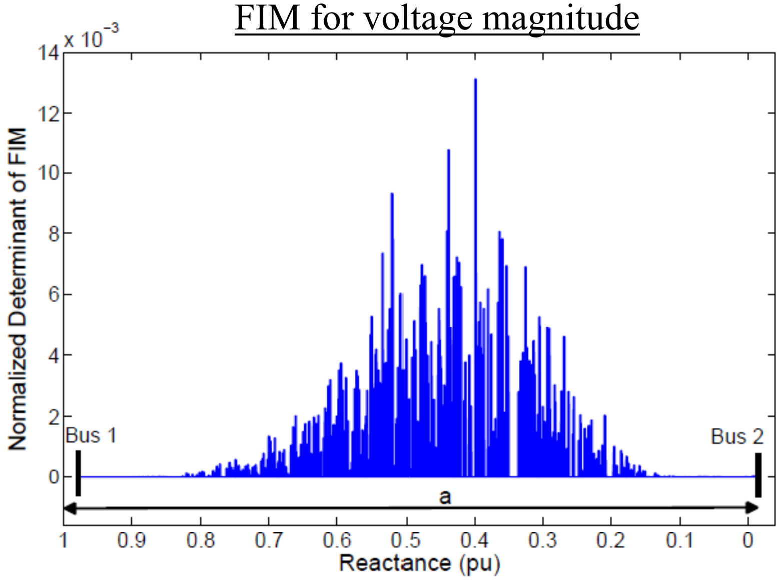
Voltage Magnitude (volts)



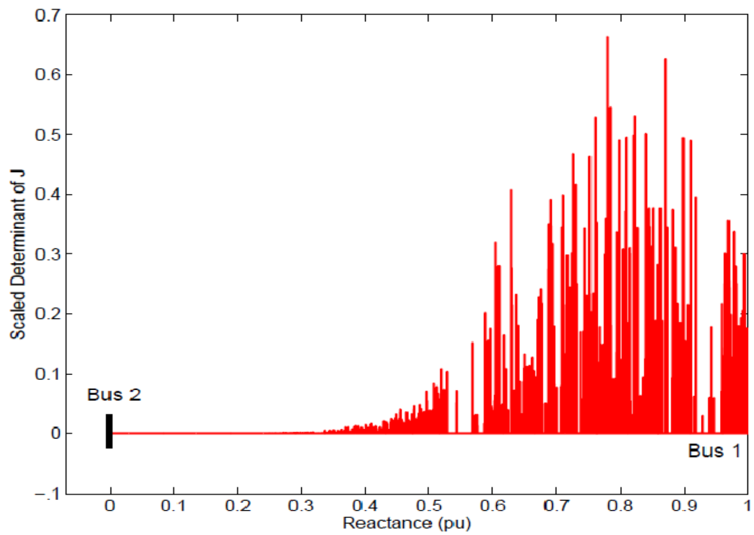
Phase Angle at Bus 1 (deg)



FIM for voltage magnitude



FIM for combined signals



Conclusions

1. A strategy for parameter estimation of swing models using noisy PMU data with a given Gaussian probability distribution

2. Estimation problem posed in terms of CR bounds

3. CRB is a function of the location of measurement

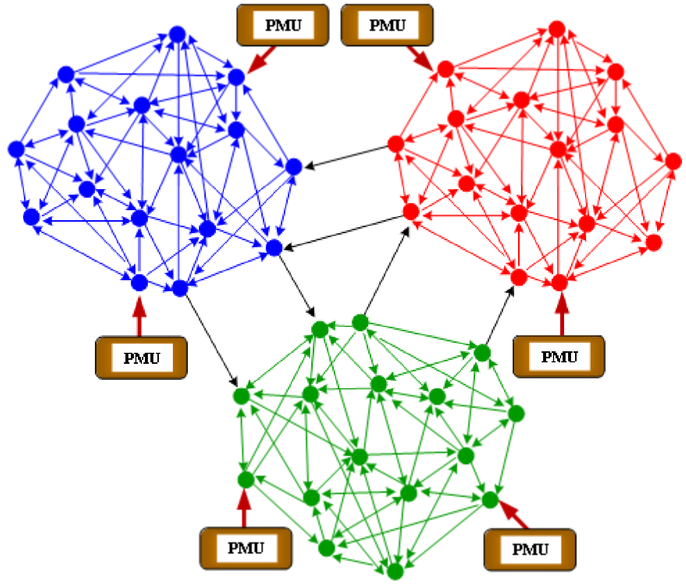
4. Not exactly a placement problem – more of an allocation problem

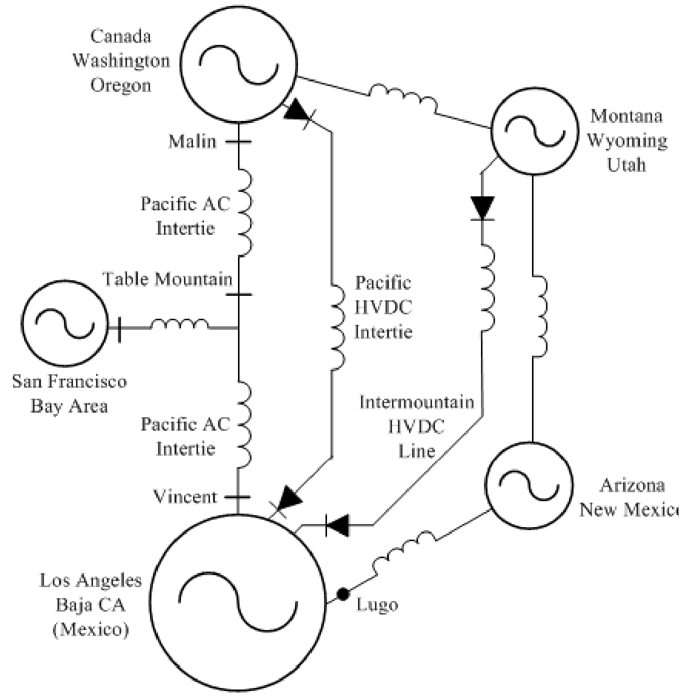
5. Even if a PMU is not placed exactly at the optimal point, the measurement at that point is still calculable from the terminal PMU's

6. Extensions: Transmission lines with intermediate voltage control, SVC feedback, TCSC (CDC 2010 - Part II, ACC 2011)

Acknowledgements: Marta Sczodrak, Bijoy Ghosh, NSF

Ongoing Phasor Research





Wide-area Monitoring

Situational Awareness (RTDMS, PGDA)
Transient Stability
Inter-area oscillations
Power-angle curves

Wide-area Modeling

How to use PMU data
for constructing area models

- So far deterministic
- Modeled for specific events
- Can be stochastic models
  (SAS may be interested)

1. Sponsor: NSF, 2010-2013 (PI: AC)
2. *CISCO: Ch & Xin*

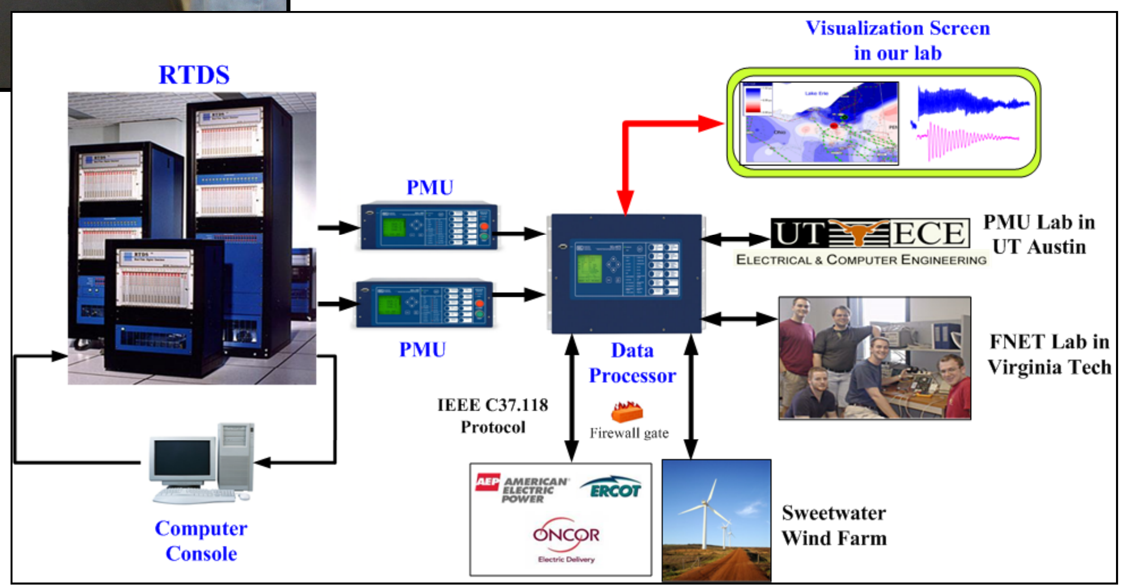
Wide-area Control

How to design controllers for
damping inter-area oscillations

- PSS
- FACTS

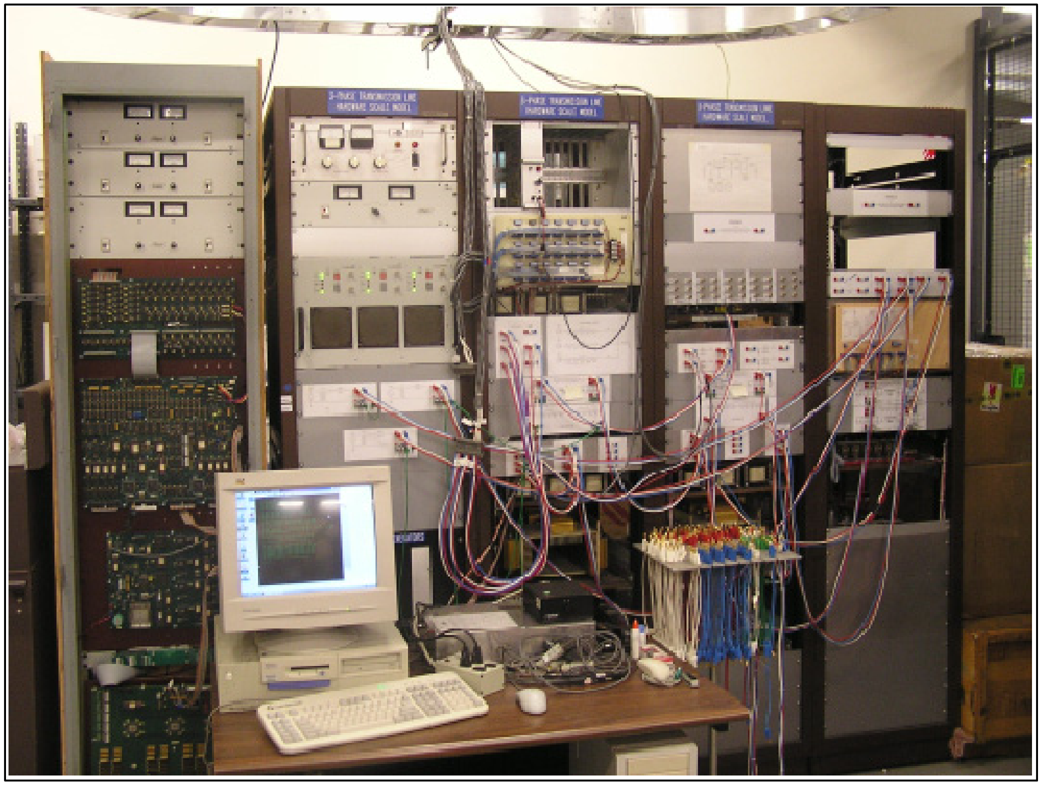
1. Sponsor: NSF, 2011-2016 (PI: AC)
2. *CPS: Mueller, Chakrabortty, Davis*
   Dvsikiotis, Ch, Michailidis

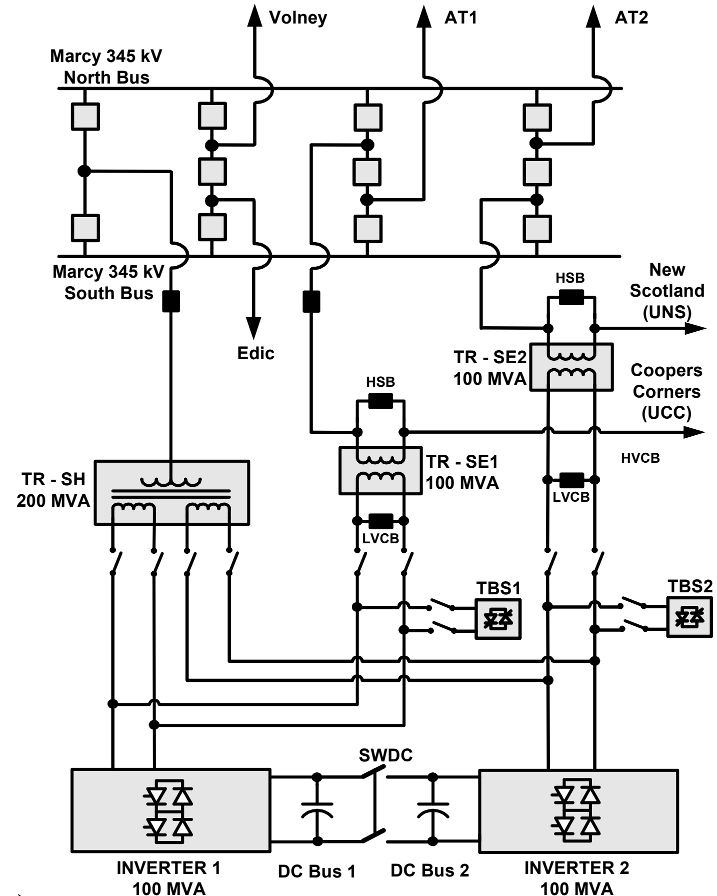
Phasor Lab



Visualization Screen
in our lab
RTDS
PMU
PMU
UT ECE
ELECTRICAL & COMPUTER ENGINEERING
PMU Lab in
UT Austin
Data
Processor
FNET Lab in
Virginia Tech
IEEE C37.118
Protocol
Firewall gate
AEP AMERICAN ELECTRIC POWER
ERCOT
ONCOR
Electric Delivery
Sweetwater
Wind Farm
Computer
Console

Transient Network Analyzer





- Joint work with Subhashish Bhattacharya (NYPA, Siemens)
- Real-time emulation of the NY grid
- Actual controllers embedded in the system
- Create fault injections at vulnerable points, measure via 3 PMUs
- Ideal test-bed for small-scale dynamic visualization within NY state
- More ambitious – controller tuning for damping control

Thank You

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